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BAYESIAN ANALYSIS OF THE CONDITIONAL CORRELATION BETWEEN STOCK INDEX RETURNS WITH MULTIVARIATE SV MODELS

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- compare the modelling ability of discrete-time Multivariate Stochastic Volatility models to describe the conditional correlations and volatilities of stock index returns

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- Bayesian comparison of different structures for the conditional covariance matrix in SV models

- Introduction
- Trivariate VAR(1)-SV models
- Empirical results
- Conclusions

References

- Jacquier E., Polson N., Rossi P., (1995), *Model and Prior for Multivariate Stochastic Volatility Models,* technical report, University of Chicago, Graduate School of Business.
- Longin F., Solnik B., (2001), *Extreme Correlation of International Equity Markets,* The Journal of Finance, Vol.56, No. 2.
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1. INTRODUCTION

Univariate Basic Stochastic Volatility Process (Clark [1973]):

$$\xi_t = \varepsilon_t \sqrt{h_t}, \qquad \varepsilon_t \sim iiN(0,1),$$

$$\ln h_t - \gamma = \phi(\ln h_{t-1} - \gamma) + \sigma_h \eta_t, \qquad \eta_t \sim iiN(0,1),$$

$$\varepsilon_t \perp \eta_s, \quad t, s \in \mathbb{Z}$$

2. TRIVARIATE VAR(1)-SV MODELS

Considered indices:

FTSE 100 ($x_{1,t}$), S&P 500 ($x_{2,t}$), WIG ($x_{3,t}$)

The vector of returns $y_t = (y_{1,t}, y_{2,t}, y_{3,t})'$, each defined by the formula $y_{j,t} = 100 \ln (x_{t,j}/x_{j,t-1})$ ($j = 1, 2, 3$), is modelled using the VAR(1) framework:

$$y_t - \delta = R(y_{t-1} - \delta) + \xi_t, \quad t = 1, 2, \ldots, T$$

where $\{\xi_t\}$ is a SV process.

More specifically

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{bmatrix} - \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \left(\begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{bmatrix} - \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} \right) + \begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \\ \xi_{3,t} \end{bmatrix}$$

where

$$\xi_t \mid \Omega_{t(i)}, \theta_i \sim N(0_{[3\times 1]}, \Sigma_t)$$

$\Omega_{t(i)}$ - the latent variable vector

<u>Prior distributions:</u>

for all elements of δ and R we assume the multivariate standardised Normal prior $N(0,I_{12})$, truncated by the restriction that all eigenvalues of R lie inside the unit circle

2.1. Stochastic Discount Factor Model – SDF

(see Jacquier, Polson and Rossi [1995])

$$\xi_t \mid \Omega_{t(1)}, \Sigma \sim N(0_{[3\times 1]}, h_t \Sigma)$$

$$\ln h_t = \phi \ln h_{t-1} + \sigma_h \eta_t, \quad \eta_t \sim iiN(0,1), \; t = 1, 2, \dots, T$$

The conditional correlation coefficients are constant over time:

$$\rho_{ij,t} = \rho = \frac{\sigma_{ij,\Sigma}}{\sqrt{\sigma^2_{ii,\Sigma}\sigma^2_{jj,\Sigma}}} \text{ for } i, j \in \{1, 2, 3\}$$

<u>Prior distributions:</u>

$\phi \sim N(0, 100)\, I_{(-1,1)}(\phi)$, $\sigma_h^2 \sim IG(1, 0.005)$, $\ln h_0 \sim N(0, 100)$,

$\Sigma \sim IW(3I, 3, 3)$

2.2. Basic Stochastic Volatility Model – BSV

$$\xi_t \mid \Omega_{t(2)} \sim N(0_{[3\times 1]}, \Sigma_t)$$

where

$$\Sigma_t = Diag(h_{1,t}, h_{2,t}, h_{3,t})$$

$$\ln h_{j,t} - \gamma_{jj} = \phi_{jj}(\ln h_{j,t-1} - \gamma_{jj}) + \sigma_{jj}\eta_{j,t}, \text{ for } j = 1, 2, 3$$

$$\eta_t \sim iiN(0_{[3\times 1]}, I_3),\ \eta_t = (\eta_{1,t}, \eta_{2,t}, \eta_{3,t})',\ \Omega_{t(2)} = (h_{1,t}, h_{2,t}, h_{3,t})'$$

<u>Prior distributions:</u>

$(\gamma_{ii}, \phi_{ii})' \sim N(0, 100I)\ I_{(-1,1)}(\phi_{ii})$, $\sigma_{ii}^2 \sim IG(1, 0.005)$, $\ln h_{i,0} \sim N(0, 100)$, $i = 1, 2, 3$

2.3. JSV Model

$$\xi_t \mid \Omega_{t(3)} \sim N(0_{[3\times 1]}, \Sigma_t)$$

where

$\Sigma_t = P \Lambda_t P^{-1}$ (spectral decomposition)

$$\Lambda_t = Diag(\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t})$$

$$\ln \lambda_{j,t} - \gamma_{jj} = \phi_{jj}(\ln \lambda_{j,t-1} - \gamma_{jj}) + \sigma_{jj}\eta_{j,t}, \text{ for } j = 1, 2, 3$$

$$\eta_t \sim iiN(0_{[3\times 1]}, I_3), \eta_t = (\eta_{1,t}, \eta_{2,t}, \eta_{3,t})', \Omega_{t(3)} = (\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t})'$$

Conditional covariance matrix:

$$P = \begin{bmatrix} \cos\kappa_3\cos\kappa_1 - \sin\kappa_1\sin\kappa_3\cos\kappa_2 & -\cos\kappa_1\sin\kappa_3 - \sin\kappa_1\cos\kappa_3\cos\kappa_2 & \sin\kappa_1\sin\kappa_2 \\ \sin\kappa_1\cos\kappa_3 + \sin\kappa_3\cos\kappa_1\cos\kappa_2 & -\sin\kappa_3\sin\kappa_1 + \cos\kappa_1\cos\kappa_3\cos\kappa_2 & -\cos\kappa_1\sin\kappa_2 \\ \sin\kappa_3\sin\kappa_2 & \cos\kappa_3\sin\kappa_2 & \cos\kappa_2 \end{bmatrix}$$

Prior distributions:

$(\gamma_{ii}, \phi_{ii})' \sim N(0, 100I)\, I_{(-1,1)}(\phi_{ii})$, $\sigma_{ii}^2 \sim IG(1, 0.005)$, $\ln\lambda_{i,0} \sim N(0, 100)$, $\kappa_i \sim U(-\pi, \pi)$, $i = 1, 2, 3$

2.4. TSV Model (see Tsay [2002])

$$\xi_t \mid \Omega_{t(4)} \sim N(0_{[3\times 1]}, \Sigma_t)$$

where

$\Sigma_t = L_t G_t L_t'$ (Cholesky decomposition)

$$L_t = \begin{bmatrix} 1 & 0 & 0 \\ q_{21,t} & 1 & 0 \\ q_{31,t} & q_{32,t} & 1 \end{bmatrix}, \; G_t = \begin{bmatrix} q_{11,t} & 0 & 0 \\ 0 & q_{22,t} & 0 \\ 0 & 0 & q_{33,t} \end{bmatrix}$$

$$\ln q_{jj,t} - \gamma_{jj} = \phi_{jj}(\ln q_{jj,t-1} - \gamma_{jj}) + \sigma_{jj}\eta_{jj,t},\ j = 1, 2, 3,$$

$$q_{ij,t} - \gamma_{ij} = \phi_{ij}(q_{ij,t-1} - \gamma_{ij}) + \sigma_{ij}\eta_{ij,t},\ j, i \in \{1, 2, 3\},\ i > j,$$

$$\eta_t = (\eta_{11,t}, \eta_{22,t}, \eta_{33,t}, \eta_{21,t}, \eta_{31,t}, \eta_{32,t})' \sim iiN_6(0_{[6\times 1]}, \mathbf{I}_6),$$

$$\Omega_{t(4)} = (q_{11,t}, q_{22,t}, q_{33,t}, q_{21,t}, q_{31,t}, q_{32,t})',\ t \in \{1, 2, \ldots, T\}$$

Conditional covariance matrix:

$$\Sigma_t = \begin{bmatrix} q_{11,t} & q_{11,t}q_{21,t} & q_{11,t}q_{31,t} \\ q_{11,t}q_{21,t} & q_{11,t}q_{21,t}^2 + q_{22,t} & q_{11,t}q_{21,t}q_{31,t} + q_{22,t}q_{32,t} \\ q_{11,t}q_{31,t} & q_{11,t}q_{21,t}q_{31,t} + q_{22,t}q_{32,t} & q_{11,t}q_{31,t}^2 + q_{22,t}q_{32,t}^2 + q_{33,t} \end{bmatrix}$$

<u>Prior distributions:</u>

$(\gamma_{ij}, \phi_{ij})' \sim N(0, 100I)\, I_{(-1,1)}(\phi_{ij})$, $\sigma_{ij}^2 \sim IG(1, 0.005)$, $\ln q_{ii,0} \sim N(0,$ $100)$ for $i, j \in \{1, 2, 3\}$ and $i \geq j$; $q_{ij,0} \sim N(0, 100)$ for $i, j \in \{1, 2, 3\}$, $i > j$

3. EMPIRICAL RESULTS

3.1. Data descriptions

Modelled indices:

FTSE 100 ($x_{1,t}$), S&P 500 ($x_{2,t}$), WIG ($x_{3,t}$)

from January 4, 1999 to December 30, 2005,

$T = 1700$ modelled observations.

Figure 1. Daily closing quotations and log returns (in percentages) of the WIG index (January 4, 1999 – December 30, 2005)

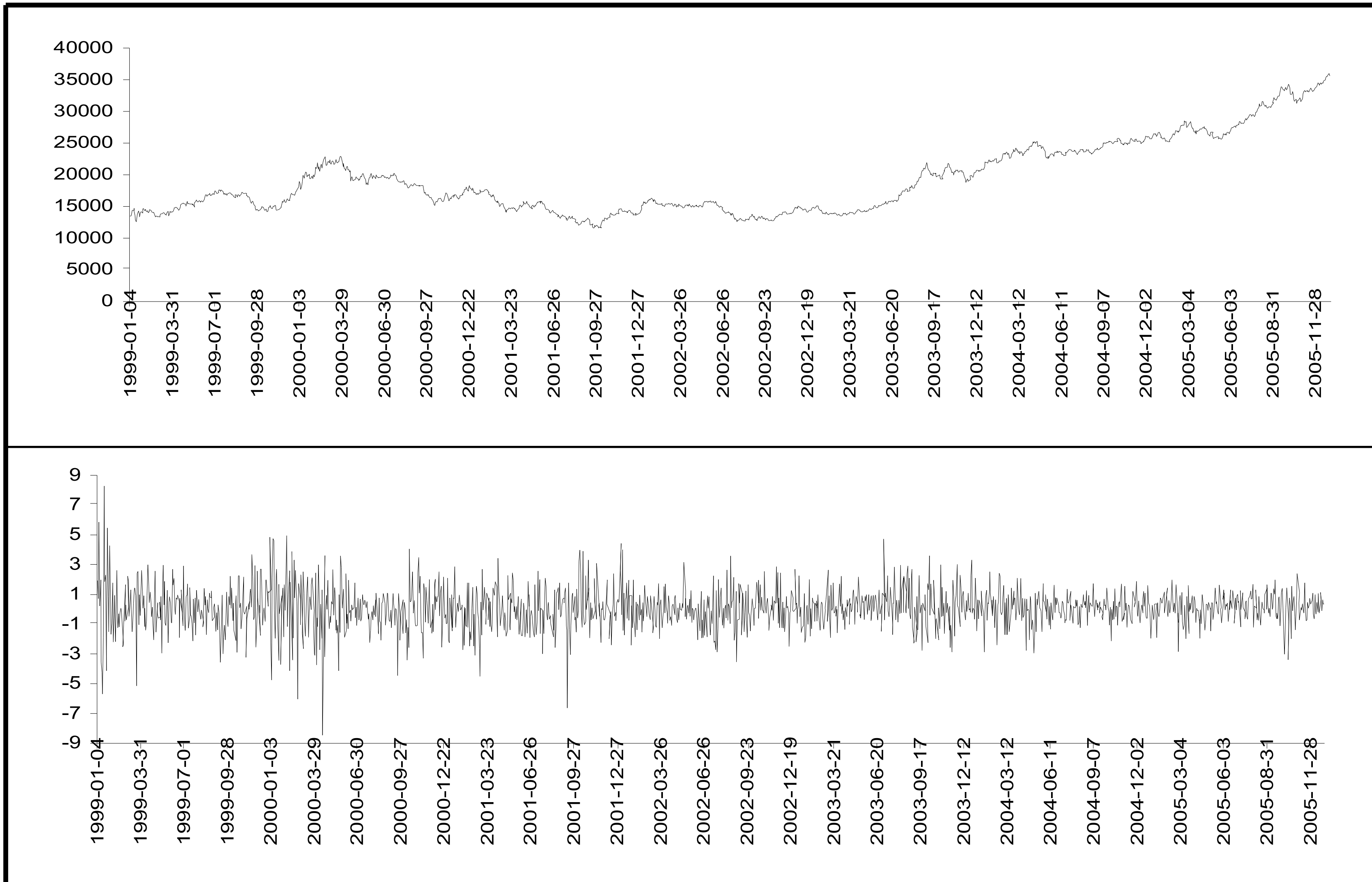

Figure 2. Daily closing quotations and log returns (in percentages) of the FTSE 100 index (January 4, 1999 – December 30, 2005)

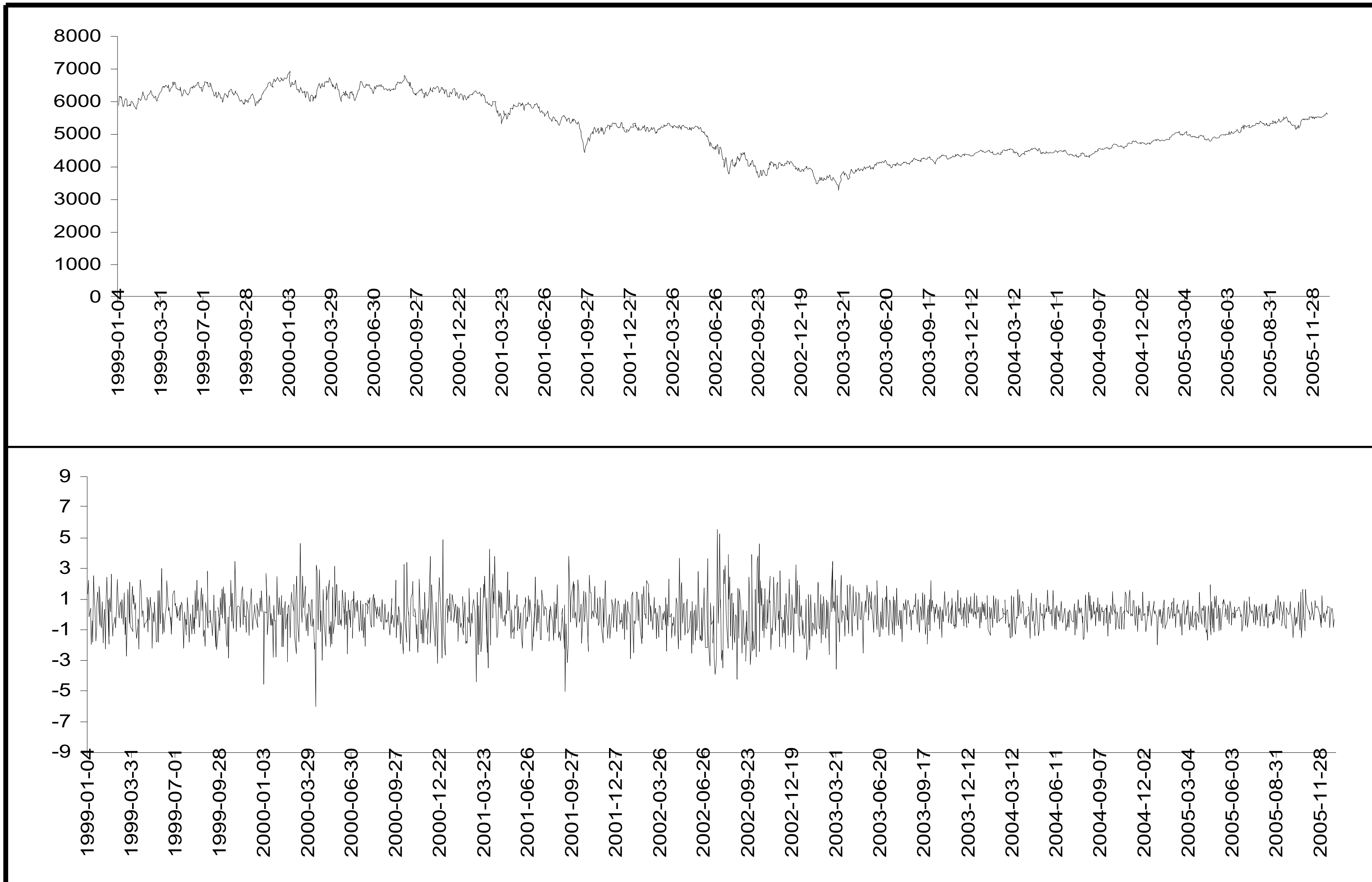

Figure 3. Daily closing quotations and log returns (in percentages) of the S&P 500 index (January 4, 1999 – December 30, 2005)

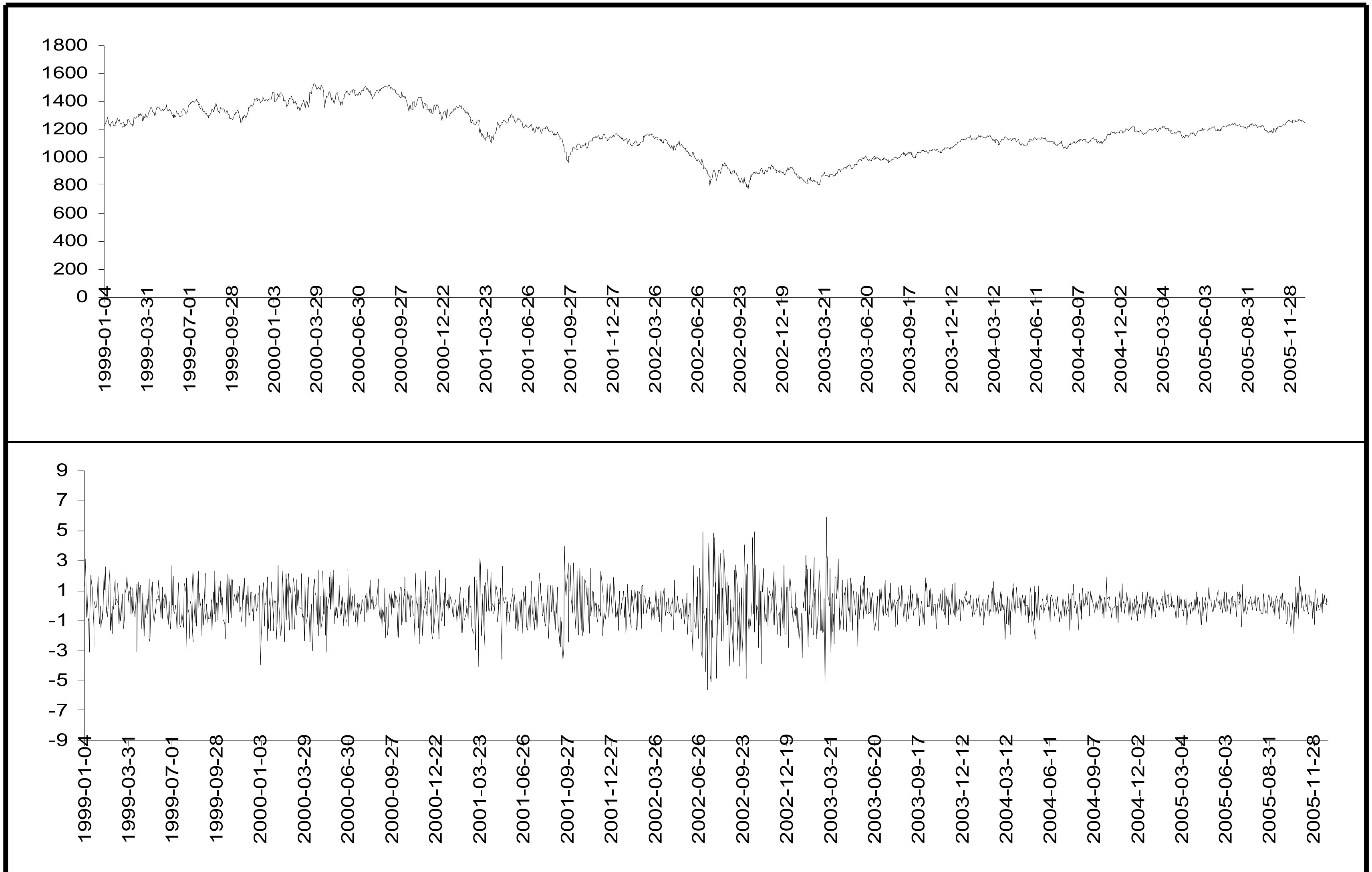

Table 1. Summary statistics of daily stock index returns from 04/01/1999 to 30/12/2005

	S&P 500	WIG	FTSE 100
Mean	0.000959	0.058247	-0.00267
Standard deviation	1.206429	1.362221	1.19227
Variance	1.45547	1.855647	1.421508
Kurtosis (excess)	1.872192	3.195033	2.471055
Skewness	0.087742	-0.06928	-0.08746
Minimum	-6.00451	-8.46784	-5.58878
Maximum	5.574432	8.291546	5.903779
Range	11.57895	16.75939	11.49256

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Table 2. Empirical correlation coefficients between daily stock index returns

	S&P 500	WIG	FTSE 100
S&P 500	1	0.18276	0.46248
WIG	0.182758	1	0.30946
FTSE 100	0.46248	0.30946	1

3.2. Bayesian model comparison

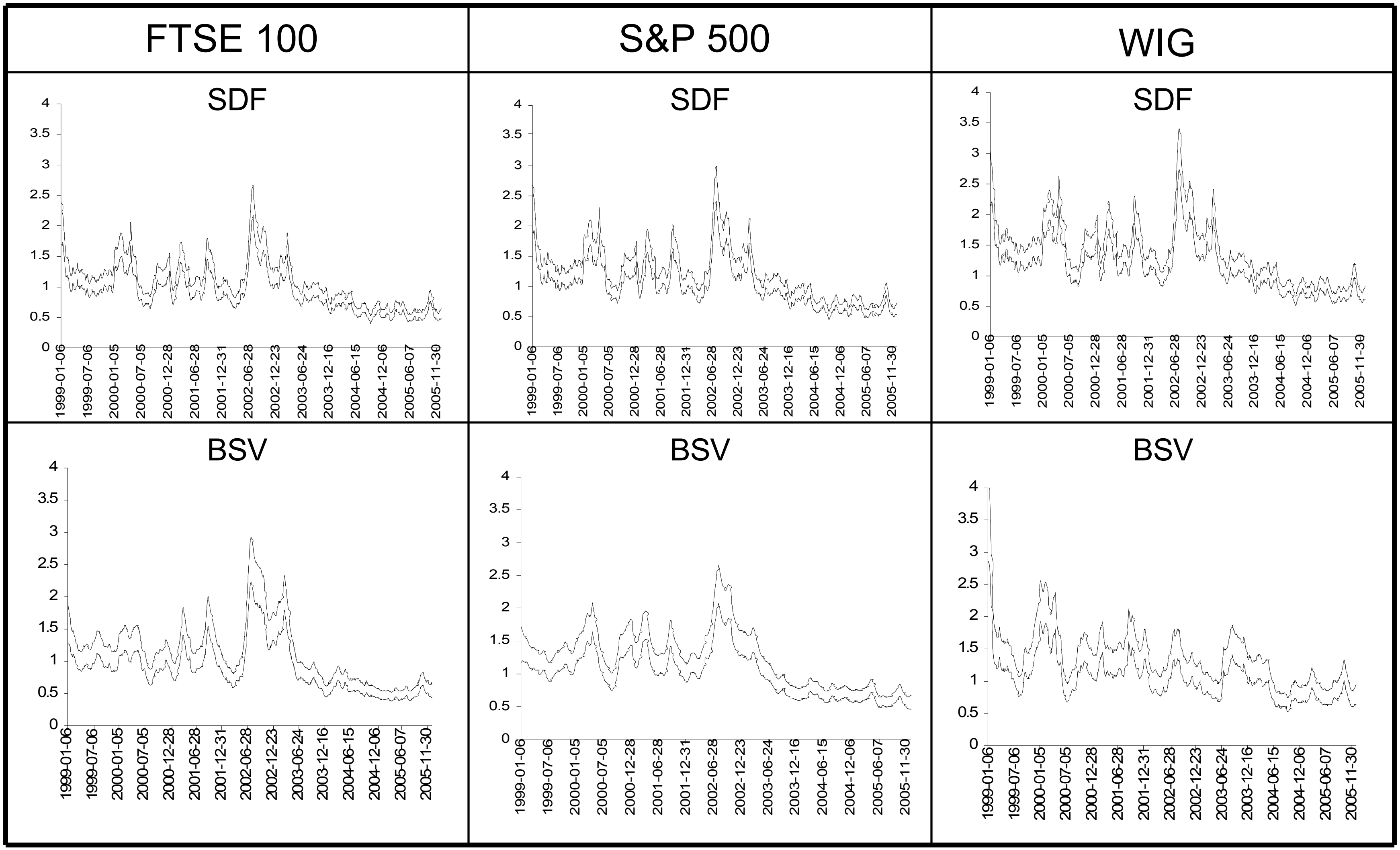
Table 3. Logs of Bayes factors in favour of VAR(1)-TSV_$_{FSW}$ model

Model	Number of latent processes	Number of parameters	$\text{Log}_{10}(B_{4,1,i})$	Rank
$M_{4,1}$ TSV_$_{FSW}$	6	12+24	0.00	1
$M_{4,2}$ TSV_$_{FWS}$	6	12+24	7.82	2
$M_{4,3}$ TSV_$_{SWF}$	6	12+24	15.55	3
$M_{4,4}$ TSV_$_{SFW}$	6	12+24	15.86	4
$M_{4,5}$ TSV_$_{WFS}$	6	12+24	17.05	5
$M_{4,6}$ TSV_$_{WSF}$	6	12+24	22.96	6
M_3 JSV	3	12+15	63.68	7
M_1 SDF	1	12+9	87.39	8
M_2 BSV	3	12+12	181.18	9

Table 4. The posterior means and standard deviations of the parameters of $M_{4,1}$ (VAR(1)-TSV$_{_FSW}$)

δ_1	δ_2	δ_3	r_{11}	r_{12}	r_{13}	r_{21}	r_{22}	r_{23}	r_{31}
0.0325 (0.0195)	0.0250 (0.0219)	0.0806 (0.0280)	-0.1993 (0.0267)	0.3223 (0.0234)	0.0149 (0.0179)	0.0175 (0.0301)	-0.0416 (0.0273)	0.0128 (0.0203)	-0.0421 (0.0252)
r_{32}	r_{33}	γ_{11}	ϕ_{11}	σ_{11}^2	γ_{22}	ϕ_{22}	σ_{22}^2	γ_{33}	ϕ_{33}
0.2861 (0.0289)	0.0241 (0.0249)	-0.5405 (1.9497)	0.9932 (0.0043)	0.0132 (0.0041)	-0.4653 (1.3939)	0.9888 (0.0070)	0.0144 (0.0064)	0.0016 (0.2745)	0.9780 (0.0089)
σ_{33}^2	γ_{21}	ϕ_{21}	σ_{21}^2	γ_{31}	ϕ_{31}	σ_{31}^2	γ_{32}	ϕ_{32}	σ_{32}^2
0.0189 (0.0079)	0.5437 (0.0248)	-0.2996 (0.1938)	0.0894 (0.0288)	0.3072 (0.0343)	0.4338 (0.3400)	0.0535 (0.0382)	0.1100 (0.0313)	0.2111 (0.4663)	0.0133 (0.0146)
$\ln q_{11.0}$	$\ln q_{22.0}$	$\ln q_{33.0}$	$q_{21.0}$	$q_{31.0}$	$q_{32.0}$				
0.8878 (0.4101)	-0.0073 (0.5532)	2.8027 (0.5895)	-0.1693 (3.5508)	-0.3580 (5.3953)	1.3863 (8.3008)				

Figure 4. Conditional standard deviations (posterior mean ± 1 standard deviation)



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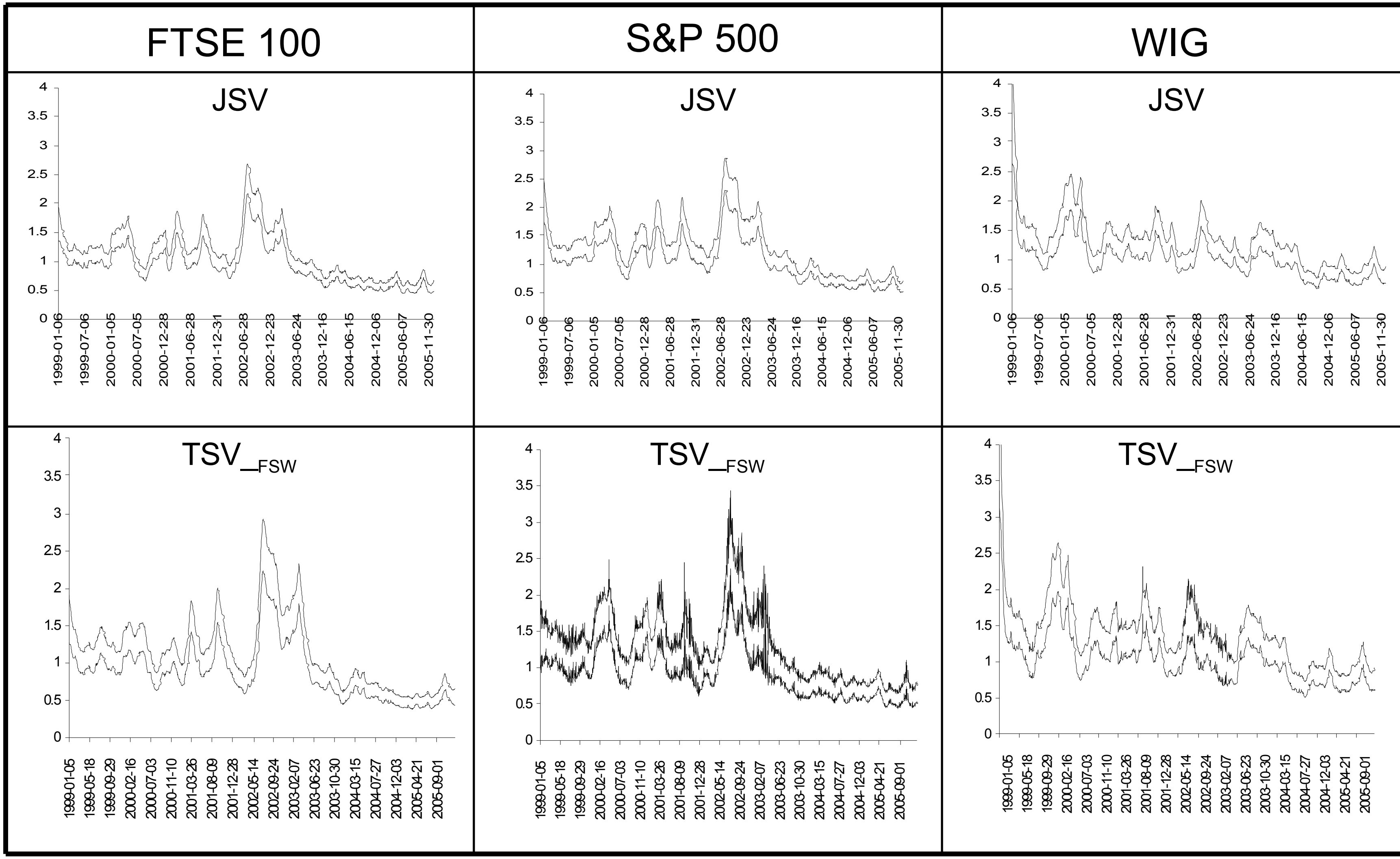
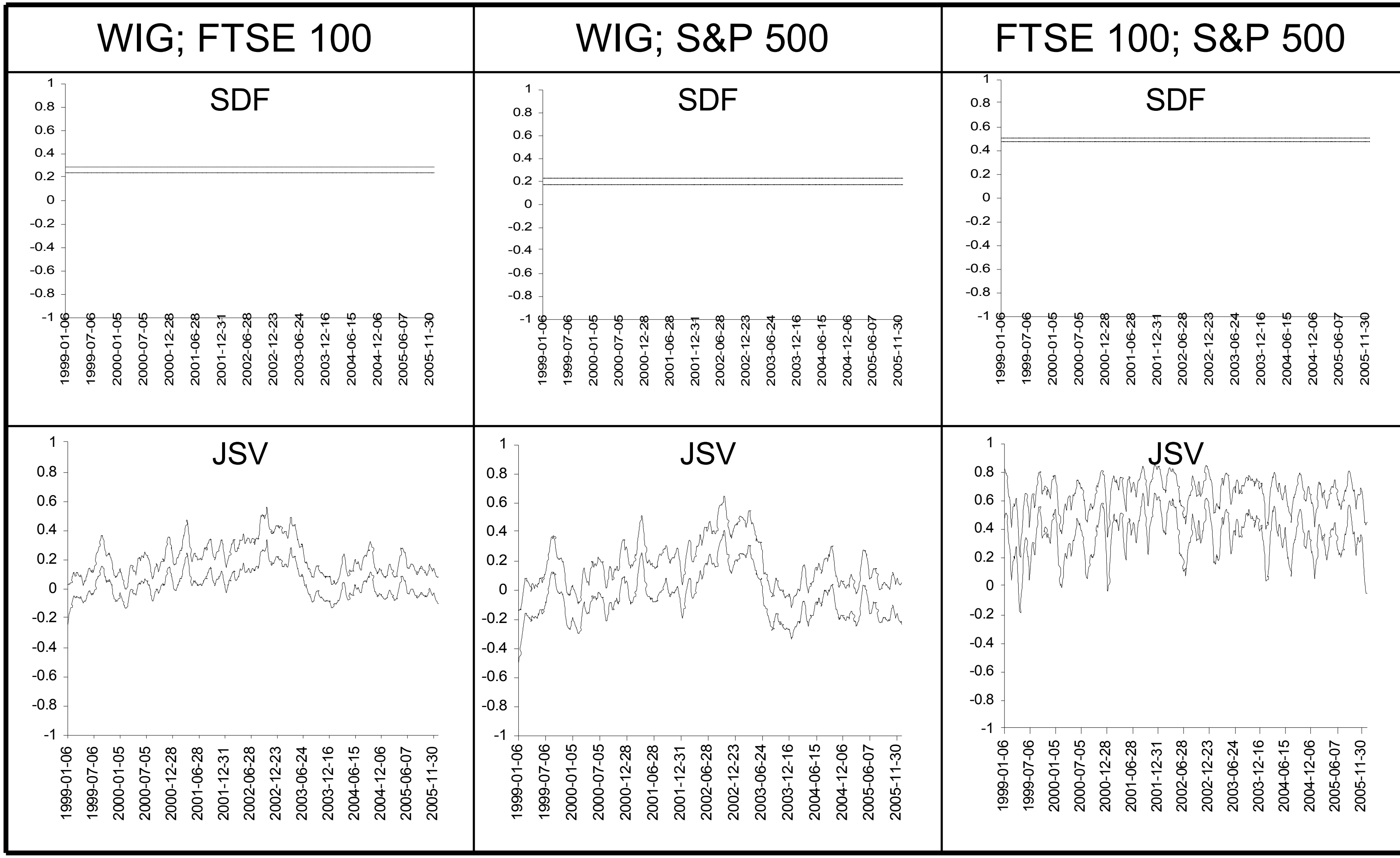


Figure 5. Conditional correlations (posterior mean ± 1 standard deviation)



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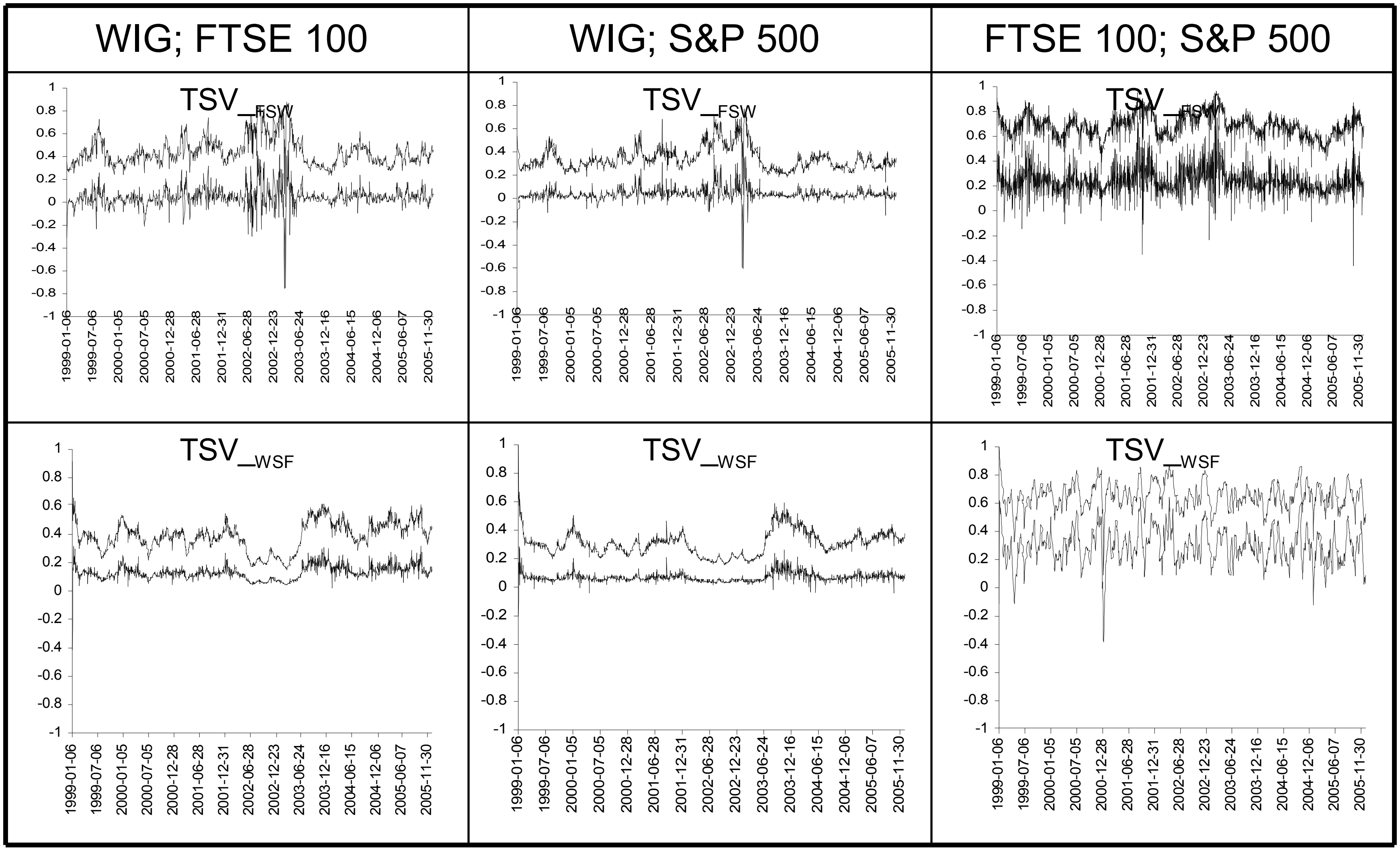

Tables 5. Correlation coefficients between the posterior means of $\rho_{ij,t}$

Corr(WIG; S&P)							
	JSV	$TSV_{\_FSW}$	$TSV_{\_FWS}$	$TSV_{\_SWF}$	$TSV_{\_SFW}$	$TSV_{\_WFS}$	$TSV_{\_WSF}$
JSV	1						
$TSV_{\_FSW}$	0.7968343	1					
$TSV_{\_FWS}$	0.371489	0.6607505	1				
$TSV_{\_SWF}$	0.8170222	0.7588796	0.4970905	1			
$TSV_{\_SFW}$	0.7854142	0.7358031	0.4931067	0.9917932	1		
$TSV_{\_WFS}$	-0.732895	-0.5842608	0.0384599	-0.4799157	-0.4428489	1	
$TSV_{\_WSF}$	-0.7579725	-0.5931888	0.0510845	-0.4916631	-0.4552313	0.9657008	1
Corr(WIG; FTSE)							
	JSV	$TSV_{\_FSW}$	$TSV_{\_FWS}$	$TSV_{\_SWF}$	$TSV_{\_SFW}$	$TSV_{\_WFS}$	$TSV_{\_WSF}$
JSV	1						
$TSV_{\_FSW}$	0.6485196	1					
$TSV_{\_FWS}$	0.639029	0.9937916	1				
$TSV_{\_SWF}$	-0.0878151	0.1609002	0.1533479	1			
$TSV_{\_SFW}$	0.7788606	0.9568819	0.9551388	0.0590551	1		
$TSV_{\_WFS}$	-0.7529246	-0.4321071	-0.4438462	0.6461421	-0.5798823	1	
$TSV_{\_WSF}$	-0.7294334	-0.4016693	-0.414338	0.6641434	-0.5655559	0.9859295	1

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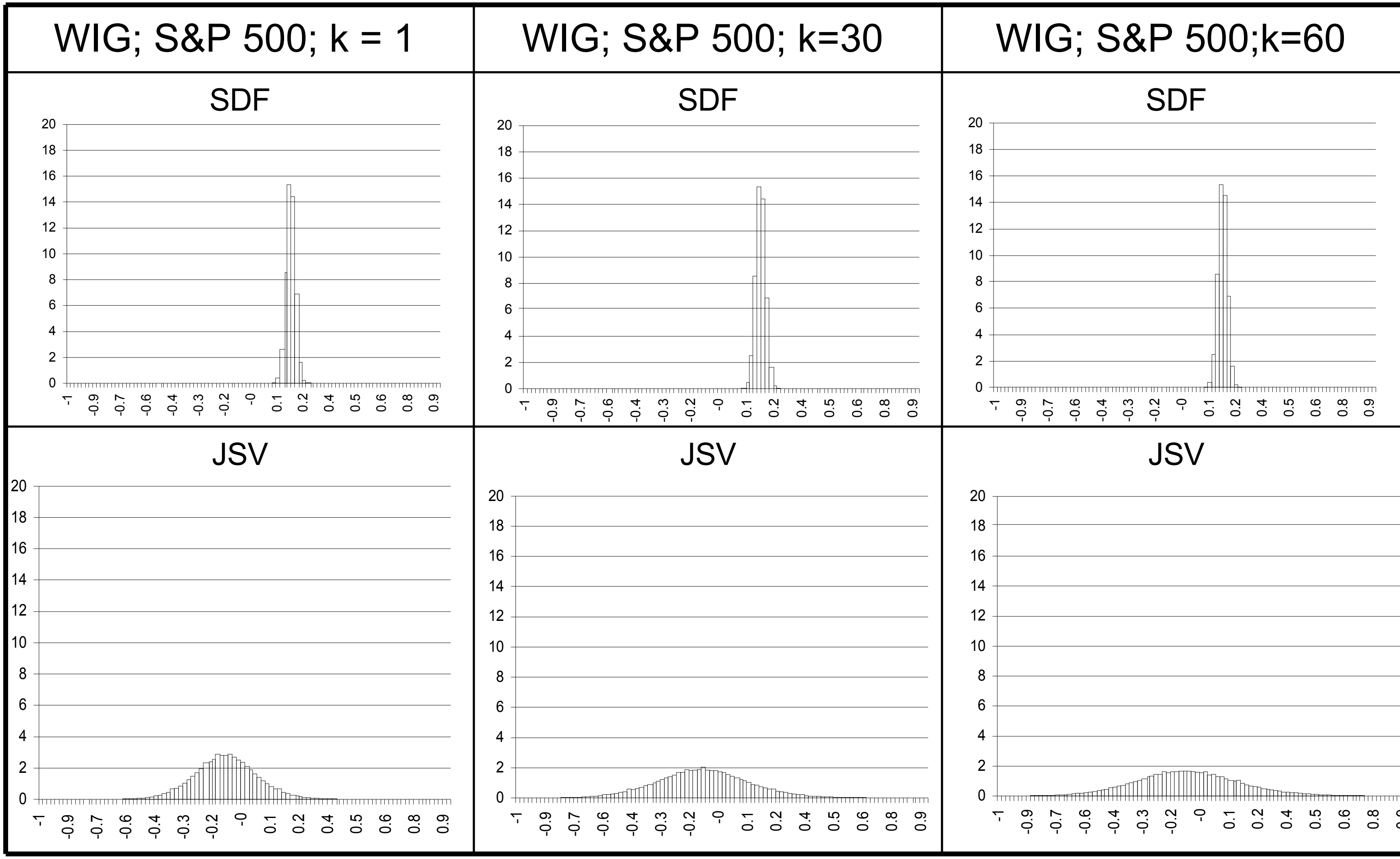
Corr(S&P; FTSE)							
	JSV	$TSV_{\_FSW}$	$TSV_{\_FWS}$	$TSV_{\_SWF}$	$TSV_{\_SFW}$	$TSV_{\_WFS}$	$TSV_{\_WSF}$
JSV	1						
$TSV_{\_FSW}$	0.3444847	1					
$TSV_{\_FWS}$	0.3360193	0.9964556	1				
$TSV_{\_SWF}$	0.8224448	0.4081398	0.3935097	1			
$TSV_{\_SFW}$	0.8406043	0.409704	0.3923261	0.9884546	1		
$TSV_{\_WFS}$	0.3510411	0.968447	0.9713927	0.398438	0.4083549	1	
$TSV_{\_WSF}$	0.8073947	0.3871765	0.3704425	0.9722056	0.972074	0.4050551	1

Table 6. Average posterior means and standard deviations of $\rho_{ij,t}$

Model VAR(1)	average $E(\rho_{ij,t}\|y, y_{(0)})$ WIG; S&P 500	average $D(\rho_{ij,t}\|y, y_{(0)})$ WIG; S&P 500	average $E(\rho_{ij,t}\|y, y_{(0)})$ WIG; FTSE 100	average $D(\rho_{ij,t}\|y, y_{(0)})$ WIG; FTSE 100	average $E(\rho_{ij,t}\|y, y_{(0)})$ S&P 500; FTSE 100	average $D(\rho_{ij,t}\|y, y_{(0)})$ S&P 500; FTSE 100
M_1 (-SDF)	0.1976	0.0239	0.2624	0.0231	0.4877	0.0192
M_2 (-BSV)	0	0	0	0	0	0
M_3 (-JSV)	0.0630	0.1183	0.1243	0.0900	0.5070	0.1540
$M_{4,1}$ (-TSV_{FSW})	0.1900	0.1533	0.2443	0.2007	0.4626	0.2221
$M_{4,2}$ (-TSV_{FWS})	0.1837	0.1606	0.2411	0.2074	0.4598	0.2231
$M_{4,3}$ (-TSV_{SWF})	0.1995	0.1601	0.2602	0.1705	0.4650	0.1852
$M_{4,4}$ (-TSV_{SFW})	0.1934	0.1661	0.2429	0.1697	0.4675	0.1810
$M_{4,5}$ (-TSV_{WFS})	0.1947	0.1156	0.2570	0.1263	0.4664	0.2083
$M_{4,6}$ (-TSV_{WSF})	0.1881	0.1203	0.2595	0.1289	0.4701	0.1812

Figure 6. Histograms of the predictive distributions of $\rho_{ij,T+k}$



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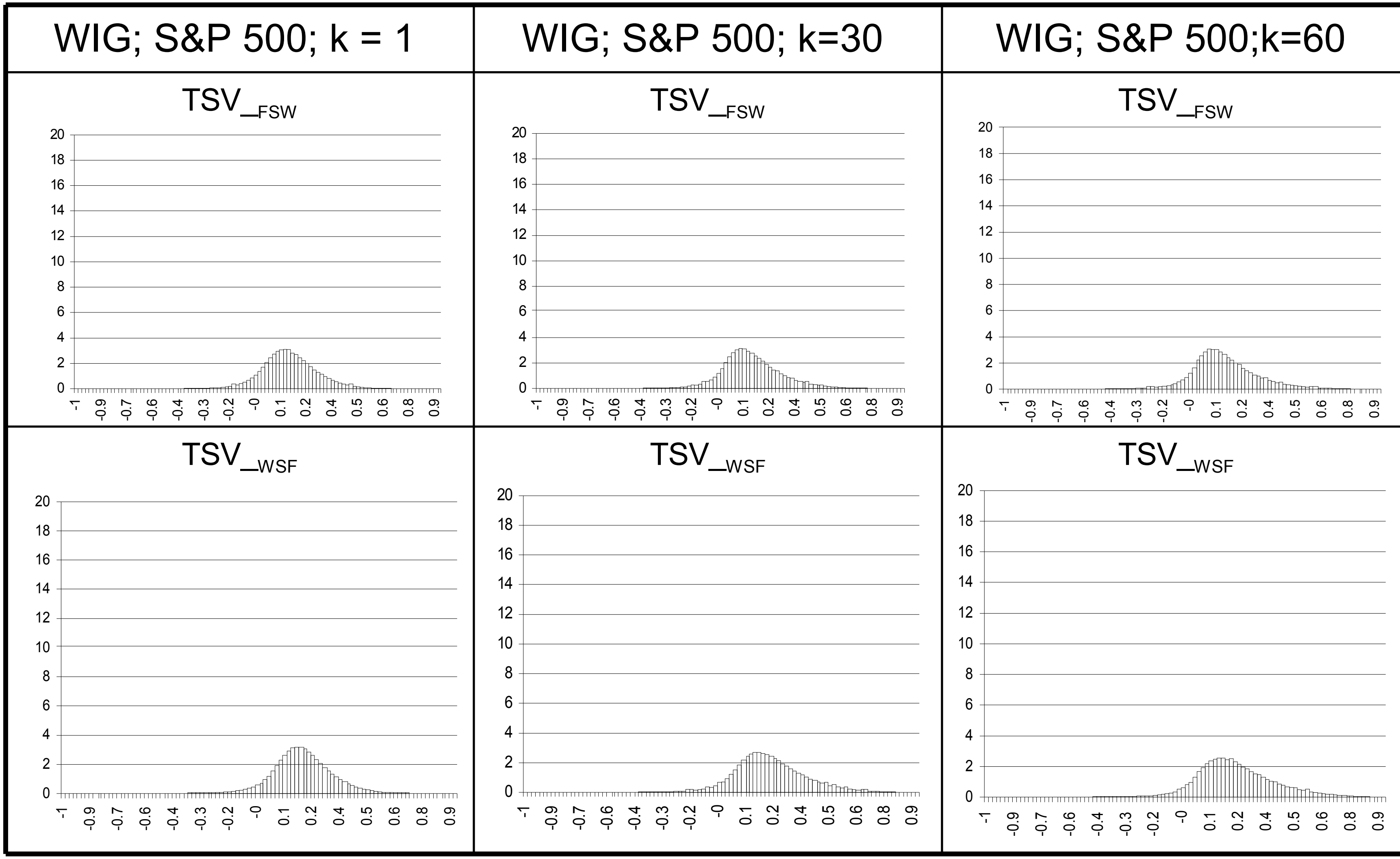

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- the data provide strong evidence against zero or constant conditional correlation coefficients between the returns
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- the VAR(1)-$TSV_{\_FSW}$ model (with as many latent variables as there are conditional variances and covariances) wins our model comparison
- the returns on WIG and FTSE indices are affected by the past movements of the U.S. stock market

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- the data provide strong evidence against zero or constant conditional correlation coefficients between the returns
- the same dynamics of all variances and covariances (SDF) seems a much better strategy than careful modeling of individual variances at the expense of zero correlations (BSV)
- the VAR(1)-TSV$_{_FSW}$ model (with as many latent variables as there are conditional variances and covariances) wins our model comparison
- the returns on WIG and FTSE indices are affected by the past movements of the U.S. stock market
- past movement of the U.K. and Poland stock markets do not significantly affect the U.S. market

Thank you!

Table 7. Quantiles of the predictive distributions of $\rho_{ij,T+k}$

		WIG; S&P			WIG; FTSE			S&P; FTSE		
	quantiles	k=1	k=30	k=60	k=1	k=30	k=60	k=1	k=30	k=60
SDF	0.05	0.159	0.159	0.159	0.226	0.226	0.226	0.456	0.456	0.456
	0.25	0.182	0.182	0.182	0.249	0.249	0.249	0.475	0.475	0.475
	0.5	0.198	0.198	0.198	0.265	0.265	0.265	0.488	0.488	0.488
	0.75	0.214	0.214	0.214	0.28	0.28	0.28	0.501	0.501	0.501
	0.95	0.237	0.237	0.237	0.302	0.302	0.302	0.519	0.519	0.519
	IQR	**0.016**	**0.016**	**0.016**	**0.0155**	**0.0155**	**0.0155**	**0.013**	**0.013**	**0.013**
JSV	0.05	-0.331	-0.45	-0.48	-0.147	-0.213	-0.232	-0.254	-0.481	-0.535
	0.25	-0.188	-0.237	-0.243	-0.071	-0.106	-0.113	0.027	-0.067	-0.077
	0.5	-0.091	-0.093	-0.081	-0.016	-0.026	-0.021	0.218	0.244	0.277
	0.75	0.005	0.052	0.086	0.047	0.075	0.101	0.393	0.513	0.571
	0.95	0.158	0.286	0.356	0.163	0.269	0.334	0.601	0.776	0.831
	IQR	**0.193**	**0.289**	**0.329**	**0.118**	**0.181**	**0.214**	**0.366**	**0.58**	**0.648**
TSV WSF	0.05	-0.01	-0.009	-0.009	0.057	0.051	0.049	-0.051	0.084	0.082
	0.25	0.13	0.132	0.131	0.194	0.179	0.175	0.197	0.295	0.285
	0.5	0.212	0.227	0.232	0.283	0.279	0.28	0.343	0.448	0.446
	0.75	0.304	0.345	0.363	0.384	0.402	0.413	0.48	0.604	0.618
	0.95	0.465	0.556	0.597	0.551	0.611	0.64	0.656	0.797	0.825
	IQR	**0.174**	**0.213**	**0.232**	**0.19**	**0.223**	**0.238**	**0.283**	**0.309**	**0.333**
TSV WFS	0.05	0.013	0.013	0.013	0.058	0.056	0.054	0.055	0.064	0.061
	0.25	0.124	0.121	0.118	0.201	0.198	0.193	0.287	0.274	0.266
	0.5	0.193	0.2	0.2	0.293	0.31	0.314	0.43	0.427	0.427
	0.75	0.273	0.299	0.307	0.396	0.446	0.467	0.565	0.587	0.602
	0.95	0.416	0.487	0.512	0.564	0.664	0.709	0.733	0.789	0.82
	IQR	**0.149**	**0.178**	**0.189**	**0.195**	**0.248**	**0.274**	**0.278**	**0.313**	**0.336**

Table 7. Quantiles of the predictive distributions of $\rho_{ij,T+k}$

		WIG; S&P			WIG; FTSE			S&P; FTSE		
	quantiles	k=1	k=30	k=60	k=1	k=30	k=60	k=1	k=30	k=60
TSV FWS	0.05	-0.071	-0.084	-0.089	-0.097	-0.106	-0.103	-0.007	0.008	0.008
	0.25	0.073	0.065	0.063	0.109	0.08	0.074	0.266	0.25	0.237
	0.5	0.164	0.158	0.158	0.231	0.193	0.185	0.432	0.425	0.418
	0.75	0.262	0.269	0.276	0.365	0.333	0.331	0.584	0.606	0.617
	0.95	0.423	0.463	0.49	0.573	0.581	0.601	0.76	0.816	0.842
	IQR	**0.189**	**0.204**	**0.213**	**0.256**	**0.253**	**0.257**	**0.318**	**0.356**	**0.38**
TSV FSW	0.05	-0.052	-0.061	-0.066	-0.087	-0.095	-0.094	-0.002	0.015	0.014
	0.25	0.082	0.071	0.068	0.11	0.081	0.075	0.274	0.256	0.241
	0.5	0.166	0.153	0.152	0.229	0.191	0.185	0.439	0.433	0.424
	0.75	0.262	0.258	0.262	0.359	0.328	0.328	0.59	0.615	0.626
	0.95	0.425	0.454	0.48	0.562	0.57	0.592	0.765	0.824	0.851
	IQR	**0.18**	**0.187**	**0.194**	**0.249**	**0.247**	**0.253**	**0.316**	**0.359**	**0.385**
TSV SFW	0.05	-0.066	-0.067	-0.066	-0.058	-0.063	-0.065	-0.046	0.051	0.049
	0.25	0.075	0.061	0.056	0.119	0.103	0.099	0.17	0.236	0.224
	0.5	0.153	0.134	0.128	0.224	0.204	0.201	0.304	0.383	0.38
	0.75	0.24	0.225	0.224	0.338	0.327	0.33	0.435	0.548	0.564
	0.95	0.401	0.411	0.427	0.526	0.545	0.564	0.614	0.765	0.802
	IQR	**0.165**	**0.164**	**0.168**	**0.219**	**0.224**	**0.231**	**0.265**	**0.312**	**0.34**
TSV SWF	0.05	-0.05	-0.054	-0.052	-0.055	-0.054	-0.056	-0.056	0.051	0.048
	0.25	0.078	0.064	0.06	0.137	0.139	0.135	0.179	0.247	0.233
	0.5	0.151	0.134	0.129	0.243	0.254	0.255	0.321	0.4	0.394
	0.75	0.233	0.222	0.223	0.356	0.384	0.394	0.458	0.564	0.576
	0.95	0.385	0.4	0.418	0.541	0.599	0.624	0.64	0.775	0.807
	IQR	**0.155**	**0.158**	**0.163**	**0.219**	**0.245**	**0.259**	**0.279**	**0.317**	**0.343**