Dynamics of wealth distribution

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Prologue


"... the distribution of wealth is too important an issue to be left to economists, sociologists, historians, and philosophers. It is of interest to everyone, and that is a good thing. The concrete, physical reality of inequality is visible to the naked eye ..."
Wealth: having it all and wanting more

January 2016 (top 1% = bottom 99%)
The number of richest who accumulated the same wealth as the poorest half of the world's population (3.5 billion people)

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>388</td>
</tr>
<tr>
<td>2011</td>
<td>177</td>
</tr>
<tr>
<td>2012</td>
<td>159</td>
</tr>
<tr>
<td>2013</td>
<td>92</td>
</tr>
<tr>
<td>2014</td>
<td>80</td>
</tr>
<tr>
<td>2015</td>
<td>62</td>
</tr>
<tr>
<td>2016</td>
<td>8</td>
</tr>
</tbody>
</table>
Inequality \( r > g \)

\( r \) = return rate on the capital

\( g \) = economy growth rate

"... When the rate of return on capital significantly exceeds the growth rate of the economy, then it logically follows that inherited wealth grows faster than output and income. People with inherited wealth need save only a portion of their income from capital to see that capital grow more quickly than the economy as a whole. ... "

Piketty’s conclusions

• Rich gets richer
• Income tax is not able to reduce the inequality
• Remedy: tax on wealth (tax on capital)
From popular-science to science


... 

Agent based-modelling

J. Angle, The surplus theory of social stratification and the size distribution of personal wealth, Social Forces 65, 293-326,(1986);

J.-P. Bouchaud and M. Mézard, Wealth condensation in a simple model of economy, Physica A 282, 536 (2000);

...
Empirical laws on wealth/income distribution

• Gibrat law
  \[ p(w) \sim \frac{1}{\sqrt{2\pi \sigma^2 w}} e^{-\frac{(\ln w - m)^2}{2\sigma^2}} \text{ for } w \sim w_* \]

• Pareto law
  \[ p(w) \sim w^{-1-\alpha} \text{ for } w \gg w_* \]

Dynamics of wealth inequality
Dynamique de l'inégalité de richesse

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State of art (milestones)

• Proportionate growth

\[ W_{\tau+1} = \lambda_{\tau} W_{\tau} \]

Gibrat law

• Kesten stochastic processes  \( \langle \lambda \rangle < 1 \)

\[ W_{\tau+1} = \begin{cases} 
\lambda_{\tau} W_{\tau}, & \text{iff } \lambda_{\tau} W_{\tau} > W_{\text{min}} \\
W_{\tau}, & \text{otherwise}
\end{cases} \]

Pareto law

• Agent models

\[ W_{i\tau+1} = \lambda_{i\tau} W_{i\tau} + \sum_j F_{ij,\tau} \]

\[ F_{ij,\tau} = - F_{ji,\tau} \]

• Taxes + redistribution
Bouchaud-Mezard model

\[ W'_{i\tau} = \lambda_{i\tau} W_{i\tau} \]

\[ W''_{i\tau} = W'_{i\tau} + \sum_j \left( J_{ij} W'_{j\tau} - J_{ji} W'_{i\tau} \right) \]

\[ W_{i\tau+1} = (1 - \beta) W''_{i\tau} + \frac{\beta}{N} \sum_j W''_{j\tau} \]

- proportionate growth
- exchange (money flow)
- state regulation (tax + redistribution)
Stable system

\[ J_{ij} = J_{ji} > 0 \]

\[ \Delta W_{i\tau} = \sum_j \left( J_{ij} W_{j\tau} - J_{ji} W_{i\tau} \right) = \sum_j J_{ij} (W_{j\tau} - W_{i\tau}) \]

Unstable system (rich-gets-richer)

\[ J_{ij} = J_{ji} < 0 \]

Risk attitude (risk aversion \( g < 1 \), risk love \( g > 1 \))

\[ \max \left| \Delta W_{i\tau} \right| = g W_{i\tau} \]
From histograms to Lorenz curve
Lorenz curve and Gini coefficient

\[ G = \frac{A}{A+B} \quad 0 \leq G \leq 1 \]
Gini coefficient by country

source: Wikipedia; by M. Tracy Hunter
Some results

- Mean-field
- Transaction rate \( J = J_{ij} \)
- Capital tax \( \beta \)
- Volatility \( \sigma^2 = \text{Var}(\log \lambda_{i\tau}) \)
- Risk neutral
Continuous time limit

\[ \frac{\partial}{\partial t} P(w, t) = \beta_* \frac{\partial}{\partial w} \left( (w - 1)p(w, t) \right) + \frac{\sigma^2}{2} \frac{\partial^2}{\partial w^2} \left( w^2 p(w, t) \right) \]

\[ \beta_* = \beta + J - \beta J > 0 \]

Stationary state

\[ P_{eq}(w) = \frac{c}{w^{1+\alpha}} \exp \left( -\frac{\alpha - 1}{w} \right) \]

\[ \alpha = 1 + 2 \frac{\beta_*}{\sigma^2} \]

\[ \alpha \approx 4.6 \]
Rich-gets-richer

Accumulated advantage

\[ \beta_* = \beta + J - \beta J < 0 \]

Matthew effect
Evolution of Gini coefficient (income tax)
Separation of wealth classes

\[ P(x) \]

\[ x = \log_{10} w \]

\[ w \]

\[ n/N \]
Distribution in urban areas of China

http://wid.world/data
Time evolution
Evolution for extremely small volatility
Varying volatility
Summary

- agent model / emergent phenomena in macroeconomy
- model of aggressive market (rich gets richer)
- capital vs income tax
- social stratification (observed in data)

Thomas Piketty

"... The inequality $r > g$ implies that wealth accumulated in the past grows more rapidly than output and wages. This inequality expresses a fundamental logical contradiction. ...

... The right solution is a progressive annual tax on capital. This will make it possible to avoid an endless inegalitarian spiral while preserving competition and incentives for new instances of primitive accumulation...

... The difficulty is that this solution, the progressive tax on capital, requires a high level of international cooperation and regional political integration. ..."
Thank you!