Probing BFKL effects with forward Drell-Yan productions

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Plan

- Drell-Yan process
- Mueller - Navelet jets
- Forward DY+jet and BFKL

Based on

KGB, L. Motyka, T.Stebel, JHEP 1812 (2018) 091

+ earlier papers

Drell - Yan process
Drell - Yan lepton pair production

- Analog of inclusive DIS in hadronic scattering

\[ d\sigma_{DY} \sim \frac{1}{Q^4} L_{\mu\nu} W_{\mu\nu} \frac{d^3 l_1}{E_1} \frac{d^3 l_2}{E_2} \]

- Cross section for lepton pair production in hadronic scattering

- Hadronic tensor

\[ W_{\mu\nu} = \int d^4x e^{iq \cdot x} \langle P_1 P_2 | J^\mu (x) J^{\nu}(0) | P_1 P_2 \rangle \]

- Two times bigger phase space and two times more structure functions.
DY structure function

- Using \((q^\mu, P^\mu, p^\mu)\) and \(q^\mu W^{\mu\nu} = 0\), four structure functions

\[
W^{\mu\nu} = (-g_T^{\mu\nu}) W_1 + (P_T^\mu P_T^\nu) W_2 - \frac{1}{2} (P_T^\mu p_T^\nu + P_T^\nu p_T^\mu) W_3 + (p_T^\mu p_T^\nu) W_4
\]

- Using photon polarization vectors \((X^\mu, Y^\mu, Z^\mu)\), helicity structure funct.

\[
W^{\mu\nu} = (X^\mu X^\nu + Y^\mu Y^\nu) W_T + (Z^\mu Z^\nu) W_L - (X^\mu Z^\nu + Z^\mu X^\nu) W_{LT} - (X^\mu X^\nu - Y^\mu Y^\nu) W_{TT}
\]

- Infinitely many choices of polarization vectors in photon rest frame where

\[
X^\mu = (0, \hat{x}) , \quad Y^\mu = (0, \hat{y}) , \quad Z^\mu = (0, \hat{z})
\]

- Each choice is called helicity frame.
Collins - Soper helicity frame \((q_\perp \neq 0)\)

- Direction of lepton momentum \(\vec{t}\) is given by spherical angles \(\Omega = (\theta, \phi)\).

\[
\frac{d\sigma_{\text{DY}}}{d^4q \ d\Omega} \sim L^{\mu\nu} W_{\mu\nu} \sim \left[ (1 + \cos^2 \theta) W_T + (1 - \cos^2 \theta) W_L + 
+ (\sin 2\theta \cos \phi) W_{LT} + (\sin^2 \theta \cos 2\phi) W_{TT} \right]
\]

- \(W_{LT}\) and \(W_{TT}\) from azimuthal angle dependence. After integration over \(\Omega\)

\[
\frac{d\sigma_{\text{DY}}}{d^4q} \sim (2W_T + W_L)
\]
QCD interpretation for $M^2 \gg \Lambda_{QCD}^2$

- In the lowest order: $q\bar{q} \rightarrow \gamma^*/Z \rightarrow l^+l^-$

- Leading twist LO DY cross section integrated over lepton angles $d\Omega$

\[
\frac{d\sigma^{DY}}{dYdM^2d^2q_\perp} \sim \sum_{i=1}^{N_f} e_i^2 \left[ q_i(x_1, M) \bar{q}_i(x_2, M) + (1 \leftrightarrow 2) \right] \delta^2(q_\perp)
\]

where $x_{1,2} = (M_\perp/\sqrt{S}) e^{\pm Y_\gamma}$. Only $W_T \neq 0$.

- No photon transverse momentum $q_\perp$ in the LO collinear approach.

- DY cross section (integrated over $q_\perp$) can be used to extract PDFs.
Origin of $q_{\perp}$ dependence

- Higher order collinear corrections (NLO is large - 50%)
  
- Soft gluon resummation $q_{\perp} \ll M$ - TMDs

- Intrinsic parton transverse momentum $q_{\perp} \sim Q$ - UPDFs.
Lam - Tung relation

\[ W_L - 2W_{TT} = 0 \]

- Analog of Callan - Gross relation in DIS: \( F_L = 0 \).
- Satisfied in NLO in contrast to Callan - Gross relation.
- Violated when quark plane \( \neq \) hadron plane

- Good indicator of non-zero parton transverse momentum.
Mueller - Navelet jets
High energy QCD describes scattering processes for which

\[ S \gg Q^2 \gg \Lambda_{QCD}^2 \]

Large logarithms \( Y = \log(S/Q^2) \) appear which must be resummed.

BFKL equation resums powers

\[ \alpha_s^n \log^n(S/Q^2) \quad \text{(LLA)}, \quad \alpha_s^{n+1} \log^n(S/Q^2) \quad \text{(NLLA)} \]

Mueller - Navelet jet production is a canonical process for BFKL studies.

Is DY process useful for high energy QCD studies?
MN jets

- Forward-backward jets separated by large rapidity $\Delta Y = \ln(\hat{S}/k_{\perp}^2) \gg 1$

- Initial partons are collinear - standard PDFs
- Gluon emissions in multi-regge kinematics

\[ y_0 \gg y_1 \gg \ldots \gg y_{n+1}, \quad |k_{i\perp}| \approx |k_{\perp}| \]

where $\Delta Y = y_0 - y_{n+1}$. Gives BFKL effects.
\[
\frac{d\sigma^{MN}}{dy_1 dy_2 dk_{1\perp}^2 dk_{2\perp}^2 d\phi} = f_{\text{eff}}(x_1, k_{1\perp}^2) \left[ \frac{C_A \alpha_s}{k_{1\perp}^2} \right] K(\vec{k}_{1\perp}, -\vec{k}_{2\perp}, \Delta Y) \left[ \frac{C_A \alpha_s}{k_{2\perp}^2} \right] f_{\text{eff}}(x_2, k_{2\perp}^2)
\]

- **BFKL kernel**

\[
K(\phi, Y) \sim \left[ l_0 + \sum_{m=1}^{\infty} 2 \cos(m(\pi - \phi)) l_m \right]
\]

where \(\phi\) is the azimuthal angle between jets and

\[
l_m(\Delta Y) = \int_0^{\infty} d\nu \exp(\omega_m(\nu) \Delta Y) \cos(\nu \ln(k_{1\perp}^2/k_{2\perp}^2))
\]

and the BFKL kernel eigenvalue in LLA

\[
\omega_m(\nu) = \tilde{\alpha}_s \left[ 2\psi(1) - \psi\left( \frac{m+1}{2} + i\nu \right) - \psi\left( \frac{m+1}{2} - i\nu \right) \right], \quad \psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}
\]
Characteristic function $\omega_m(\nu)$

$m = 0$ (red curve) gives dominant behaviour in $Y$ (energy)

$$K(\Delta Y) \sim \exp(0.25\Delta Y) \sim S^{0.25}$$

$m > 1$ (blue curves) give azimuthal angle dependence

Angular decorrelation - jets are no longer back-to-back due to gluon emissions
Angular decorrelation

- Cross section measured by CMS collaboration

\[
\frac{1}{\sigma^{MN}} \frac{d\sigma^{MN}}{d\phi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{m=1}^{\infty} \cos(m(\pi - \phi)) \langle \cos(m(\pi - \phi)) \rangle \right\}
\]

- Back-to-back jets: \( \langle \cos(m(\pi - \phi)) \rangle = 1 \)

\[
\frac{1}{\sigma^{MN}} \frac{d\sigma^{MN}}{d\phi} = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im(\pi - \phi)} = \delta(\pi - \phi)
\]

- Red curves from BFKL calculations of Ducloué, Szymanowski and Wallon.
DY + jet
Forward Drell - Yan and backward jet

\[ \Delta Y_P = \ln \left( \frac{z(1 - z) x_1 x_2 S}{M^2(1 - z) + q_T^2 + z(k_{1\perp}^2 - 2 k_{1\perp} \cdot q_T)} \right), \quad z = \frac{p_{J\perp} M_{\perp}}{x_1 x_2 S} e^{\Delta Y_{\gamma J}} \]

-\( \Delta Y_P \) is an argument of the BFKL kernel while \( \Delta Y_{\gamma J} \) is measured

- Theoretical \( \Delta Y_P \) depends on measured \( \Delta Y_{\gamma J} \)
For DY+jet helicity structure functions for $\lambda = T, L, TT, LT$

$$W_{\lambda} \equiv \frac{d\sigma_{\lambda}^{DY}}{d^4q} \quad \rightarrow \quad W_{\lambda}^{DY+j} \equiv \frac{d\sigma_{\lambda}^{DY+j}}{d^4q \, d^2p_{J\perp}}$$

and $d^4q = \frac{1}{2} dY_{\gamma J} dM^2 dq_{\perp}^2 \, d\phi_{\gamma J}$.

Explicitly

$$W_{\lambda}^{DY+j} = \frac{4\alpha_{em}^2 \alpha_s^2}{(2\pi)^4} \frac{1}{M^2 p_{J\perp}^2} \int dx_1 \int dx_2 \, f_{q\bar{q}}(x_1, M_{\perp}) \, f_{\text{eff}}(x_2, M_{\perp}) \, \theta(1 - z) \times \int \frac{d^2k_{1\perp}}{k_{1\perp}^2} \Phi_{(\lambda)}^{\gamma J}(q_{\perp}, k_{1\perp}, z) \, K(\vec{k}_{1\perp}, -\vec{p}_{J\perp}, \Delta Y_P)$$

where $\Phi_{(\lambda)}^{\gamma J}$ is the LO photon/jet impact factor and $K$ is the BFKL kernel.
BFKL kernel eigenvalues $\omega_m(\nu)$

- BFKL kernel with consistency condition (CC) (part of NLLA corrections)
- Eigenvalues $\omega = \omega_m(\nu)$ from equation

$$\omega = \bar{\alpha}_s \left[ 2\psi(1) - \psi\left(\frac{\omega + m + 1}{2} + i\nu\right) - \psi\left(\frac{\omega + m + 1}{2} - i\nu\right) \right]$$
Angular decorrelation in DY+jet process

- Cross section integrated over lepton angles - combination $T + \frac{1}{2} L$

$$\sigma(\phi_{\gamma J}) \equiv \frac{d(\sigma_T + \sigma_L/2)}{dp_{j\perp}^2 \left( dM^2 d\Delta Y_{\gamma J} dq_{\perp}^2 d\phi_{\gamma J} \right)}$$

- For LHC energy $\sqrt{S} = 13$ TeV and

$$p_{J\perp} = 30 \; \text{GeV}, \quad M = 35 \; \text{GeV}, \quad \Delta Y_{\gamma J} = \Delta Y_{MN} = 7$$

- Plotted ratio

$$\frac{\sigma(\phi_{\gamma J})}{\sigma(0)}$$

- $\gamma J$ decorrelation - flat distribution in $\phi_{\gamma J}$
Angular decorrelation in DY+jet process

▶ Stronger decorrelation for DY+j than for MN jets.
Angular decorrelation as a function of $\Delta Y_{\gamma J}$

- Given in terms of mean cosines $\langle \cos(m\phi_{\gamma J}) \rangle$
- $\gamma J$ decorrelation $\Rightarrow \langle \cos(m\phi_{\gamma J}) \rangle < 1$

$q_{\perp} = p_{l \perp} = 25 \text{ GeV}$  $p_{J \perp} = 30 \text{ GeV}$  $M = 35 \text{ GeV}$

- Stronger decorrelation for DY+j than for MN jets.
DY+ jet helicity structure functions

- From angular dependence of DY lepton pair helicity structure functions for $DY + j$

\[
\begin{align*}
A_0 &= \frac{W_L}{W_T + W_L/2}, & A_1 &= \frac{W_{LT}}{W_T + W_L/2}, & A_2 &= \frac{2W_{TT}}{W_T + W_L/2}
\end{align*}
\]

- Lam-Tun relation: $A_0 - A_2 = 0$

- Additional information about BFKL effects.
Total cross section

\[ \frac{d\sigma}{dM} [\text{pb/GeV}] \]

- \( q_\perp > 10 \text{ GeV} \)
- \( p_{J\perp} > 20 \text{ GeV} \)
- \( y_\gamma < 4 \)
- \( y_J < 4.7 \)
- \( \Delta Y_{\gamma J} > 4 \)
▶ DY + jet process was proposed to test BFKL effects.
▶ More observables than for MN jets and cleaner experimental signal.
▶ Stronger angular decorrelation than for MN jets.
▶ Helicity structure functions are sensitive to BFKL dynamics.
Best wishes for all women today!