MAXIMUM LIKELIHOOD METHOD
CONFIDENCE INTERVALS

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1. Eight trials are conducted of a given system with the following results: **S, F, S, F, S, S, S, S** (**S** – success; **F** – failure). What is the maximum likelihood estimate of p, the probability of successful operation?

2. Consider the geometric distribution: probability that an event A, having the probability $\mathcal{P} = p$ will occur after $x - 1$ trials in which A does not occur:

$$f(x;p) = p(1-p)^{x-1}; \qquad x = 1, 2, \ldots$$

Suppose a series of n such experiments are carried out and let $x_1, x_2, \ldots, x_n$ denote the number of trials before A occurs in each experiment. Find an estimate for p using the method of maximum likelihood.

CONFIDENCE INTERVALS

1. The average zinc concentrations recovered from a sample of zinc measurements in 36 different locations on a river was found to be 2.6 grams per millilitre. Find the 95% and 99% confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation σ is 0.3 grams per millilitre.

   Hint: The sample is big enough to use the appropriate quantiles of the standard normal distribution. Anyway, we have no other choice – we have no data to estimate the population standard deviation, so we consider σ as being known for good.

2. **Adequate sample size.** If $\bar{x}$ is used as an estimate of μ one can (roughly) say that: I am (1-α)100% confident that the error will not exceed $E = u_{1-\alpha/2}\sigma/\sqrt{n}$. So, reshaping this statement: If $\bar{x}$ is used as an estimate of μ I am (1-α)100% confident that the error will not exceed a specified amount E when the sample size is:

$$n = \left(\frac{u_{1-\alpha/2}\sigma}{E}\right)^2.$$

   Find how large a sample is required in the preceding example if we want to be 95% confident that our estimate of μ is off by less than 0.05.

3. The contents of 7 similar containers of sulphuric acid are: 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 litres. Find the 95% confidence interval for the mean of all such containers, assuming an approximate normal distribution.

4. In a random sample of $n = 500$ families owning a TV set in the city of Hamilton (Canada) it was found that $x = 340$ subscribed to the HBO (a cable TV company). Find a 95% confidence interval for the actual proportion of families in this city who subscribe to HBO.

   Hint: This is a Bernoulli-type problem, but the Bernoulli distribution tends to normal if the size is adequate (it is). The point estimate of "probability of success" (if one considers it a success to be an HBO subscriber) is $p = x/n = 340/500 = 0.68$. The variance of this average value is the variance of x, $VAR(x) = npq$ divided by n^2 (why?), i.e. pq/n where $q = 1 - p$. Using our favourite normal distribution quantiles (1.96) we get the interval (0.64,0.72).

5. An electrical firm manufactures light bulbs that have a length of life approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

   How large a sample is needed if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?