CONFIDENCE INTERVALS, STATISTICAL HYPOTHESES

1. A certain type of storage battery lasts on the average 5.0 years; with a standard deviation of 0.8 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 4.3 years.

$$Z = \frac{4.3 - 5}{0.8} = -0.875; \quad \mathcal{P}\left(X < 4.3\right) = \mathcal{P}\left(Z < -0.875\right) \approx 0.191.$$

2. A process yields 10% defective items. If 100 items are randomly selected what is the probability that the number of defectives

   (a) exceeds 13; (i.e greater than 12.5);

   (b) is less than 8 (i.e. less than 7.5).

   Hint; Use Bernoulli-to-Normal approximation. $E(X) = ?; \quad \sigma^2 = ?$

   $E(X) = 100 \times 0.1 = 10$ $\sigma^2 = npq = 100 \cdot 0.1 \cdot 0.9 = 9$; $\sigma = 3$.

$$Z = \frac{X - E(X)}{\sigma}$$

   – this variable is normally distributed (a) $\mathcal{P}\left(X > 13\right) = \mathcal{P}\left(X > 12.5\right) = \mathcal{P}\left(Z > 2.5/3\right) = \text{tables} \approx 0.2$

   (b) $\mathcal{P}\left(X < 8\right) = \mathcal{P}\left(X < 7.5\right) = \mathcal{P}\left(Z < -2.5/3\right) = \text{tables} =\approx 0.2$

3. The average zinc concentration recovered from a sample of zinc measurements in 25 different locations was found to be 3.2 grams per milliliter. Find the 95% confidence interval for the mean zinc concentration in the river. Assume that the population standard deviation is 0.4 grams per milliliter.

   $\bar{x} = 3.2$

$$3.2 - q\frac{0.4}{\sqrt{25}} < \mu < 3.2 + q\frac{0.4}{\sqrt{25}},$$

   where q is appropriate quantile of the normal distribution;for a 95% confidence interval $q_{0.975} = 1.96$. The rest of calculations is left to you.

4. A manufacturing process produces cylindrical component parts for the automotive industry. It is important that the process produce parts having a mean of 15 millimeters. The engineer involved conjectures that the population mean (i.e., the expected value for the whole production) is 15.0 mm. An experiment is conducted in which 400 parts produced by the process are selected randomly and the diameter measured on each. It is known that the population standard deviation (i.e. the standard deviation characteristic for every individual part) σ is 0.25. The experiment gives a sample average diameter $\bar{x} = 15.02$ mm. Does this sample information appear to support or refute the engineer's conjecture?

   Hint: In other words, we test the H0 hypothesis $\mu = 15$. Assume $\alpha = 0.05$ and H1: $\mu \neq 15$.

$$Z = \frac{15.02 - 15.00}{\sigma/\sqrt{n}} = \frac{15.02 - 15.00}{0.25/\sqrt{400}} = 1.6$$

   the critical (rejection) region for rejecting H0 are the symmetric two-tails of the normal distribution containing each 2.5% of the area under the normal graph: $q_{0.025}$ and $q_{0.975}$ ∓ 1.96. The obtained Z-value is NOT within these two tails (not very far, though) – we have no reason to reject H0.

5. In order to estimate the mean braking distance for a car (at a given speed, 40kms/h) 12 trials were run, giving the results (in meters):

   17,8 19,2 22,0 21,4 19,8 21,2 20,7 18,7 21,1 17,9 20,6 19,6

   Determine a 95% confidence interval for the braking length.

   Hint calculate the mean: $\bar{x} =$ and the standard deviation estimate S^*:

$$S^{*2} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = 1.93; \qquad S^* = 1.39$$

   The two-sided interval uses the Student's $t_{0.975}(n-1)$ quantile. Ask your teacher for the adequate table.

$$\bar{x} - t_{0.975(n-1)}\frac{S^*}{\sqrt{n}} < \bar{x} + t_{0.975(n-1)}\frac{S^*}{\sqrt{n}}$$

or

$$20 \pm 2.2 \frac{1.39}{\sqrt{12}} = 20 \pm 0.883$$

6. We define a fair die to mean that each face is equally likely; thus, there are six categories, and the null hypothesis we wish to test is:

$$H0 : p_i = 1/6, \quad i = 1, 2, \ldots, 6$$

To determine if a die is fair, as opposed to the alternative that it is not, a die was thrown $n = 60$ times, with the results displayed in the table. Complete columns 3 and 4, compute the χ^2 statistic, and test the null hypothesis at the level $\alpha = 0.05$.

Face	Observed frequency, y_i	Expected frequency, $n \times p_i$	$y_i - np_i$
1	10		
2	9		
3	11		
4	11		
5	8		
6	11		

The necessary $\chi^2_{?;0.95}$ quantile – ask the teacher. calculated χ^2 statistic is $0.8 < \chi^2_{5;0.95} = 11.07$.; consequentlly the null hypothesis is not rejected.