PARAMETRIC STATISTICAL TESTS
TESTS concerning the EXPECTED VALUE of RV

STATISTICAL HYPOTHESIS: $H_0 : E\{X\} \equiv \mu = \mu_0$

— its "adversary" ALTERNATIVE HYPOTHESIS can be one of the following 3:

a) $H_1 : \mu = \mu_1 < \mu_0$;
b) $H_1 : \mu = \mu_1 > \mu_0$;
c) $H_1 : \mu = \mu_1 \neq \mu_0$.

TEST STATISTIC T is

1. case **a**: σ known; (sample size big enough, so $\sigma \approx S$ (S^*); or σ given as an estimate of the single–measurement error). We form:

$$Z = \frac{\bar{X} - \mu_0}{\sigma}\sqrt{n}$$

   If $H_0 : \mu = \mu_0$ is true the RV Z obeys the distribution $N(0, 1)$

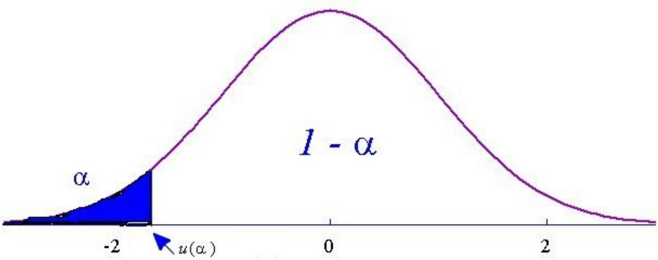
2. case **b**: σ unknown, BUT the S-statistic value is known

$$t = \frac{\bar{X} - \mu_0}{S}\sqrt{n - 1}$$

   If $H_0 : \mu = \mu_0$ is true the RV t obeys the Student's distribution $t(\nu)$, where $\nu = n - 1$ is the number of the degrees of freedom.

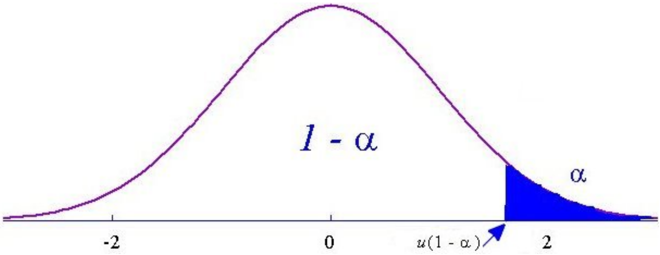
σ known; $T = U$ — normal distribution

$H_1 : \mu = \mu_1 < \mu_0$; $U_{cr} = \big(-\infty, u(\alpha) \big)$ — LEFT–TAILED (LEFT–SIDED)

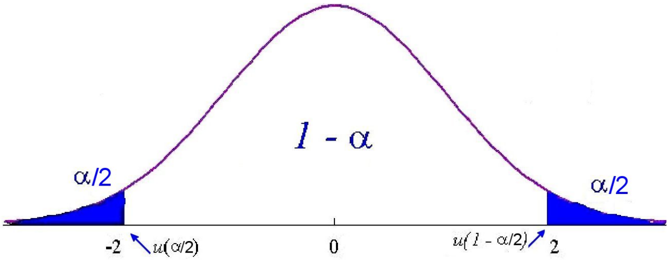


σ known; $T = U$ — normal distribution

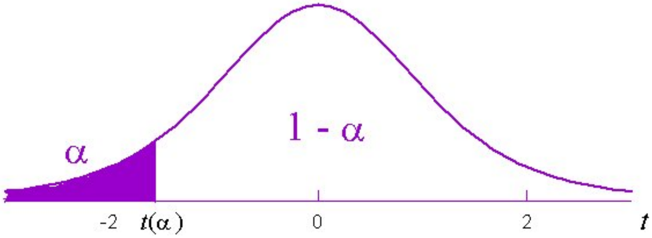
$H_1 : \mu = \mu_1 > \mu_0$; $U_{cr} = \left(u(1-\alpha), +\infty\right)$ — RIGHT–TAILED (RIGHT–SIDED)



$H_1 : \mu = \mu_1 \neq \mu_0$; $U_{cr} = \Big(-\infty, u(\alpha/2)\big) \bigcup \big(u(1-\alpha/2), +\infty\Big)$ — TWO–TAILED

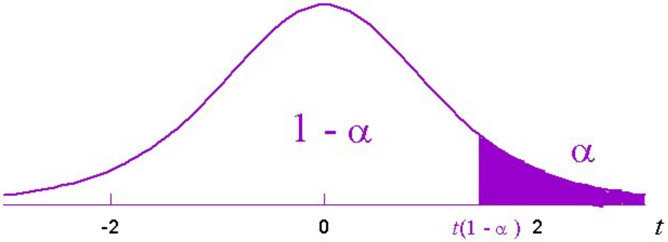


$$H_1 : \mu = \mu_1 < \mu_0 \; ; \quad U_{cr} = \Big(-\infty, t(\alpha, n-1) \Big) \text{ — LEFT–TAILED}$$



σ estimated; $T = t$ — follows Student's distribution

$H_1 : \mu = \mu_1 < \mu_0$; $U_{cr} = \big(t(1-\alpha, n-1), +\infty\big)$ — RIGHT–TAILED



$H_1 : \mu = \mu_1 < \mu_0$; $U_{cr} =$
$\big(-\infty, t(\alpha/2, n-1)\big) \bigcup \big(t(1-\alpha/2, n-1), +\infty\big)$ — TWO–TAILED