PARAMETRIC STATISTICAL TESTS
testing the equality of
VARIANCES

the so-called FISCHER TEST

he investigated RV X follows — in 2 populations — the distributions

$$N(\mu_1, \sigma_1), \quad N(\mu_2, \sigma_2)$$

(all four parameters: $\mu_1, \mu_2, \sigma_1, \sigma_2$ are unknown)
HYPOTHESIS to be VERIFIED: $H_0 : \sigma_1 = \sigma_2 \equiv \sigma$

As usually, we form two independent samples:

$$\text{sample No.1:} \quad X_1, X_2, \ldots, X_{n_1}$$

$$\text{sample No.2:} \quad Y_1, Y_2, \ldots, Y_{n_2}$$

we calculate the means and the mean-square deviations from the means:

$$S_X^{*2} = \frac{1}{n_1 - 1} \sum_{k=1}^{n_1} (X_k - \bar{X})^2 \quad S_Y^{*2} = \frac{1}{n_2 - 1} \sum_{k=1}^{n_2} (Y_k - \bar{Y})^2$$

Now, the RATIO of the unbiased variance estimators is the so–called

STATISTIC $\mathcal{F}$:

$$\boldsymbol{\mathcal{F}} = \frac{S_X^{*2}}{S_Y^{*2}} = \frac{\dfrac{n_1}{n_1 - 1} S_X^2}{\dfrac{n_2}{n_2 - 1} S_Y^2}$$

this statistic (RV) follows a standard distribution:
it's the so–called SNEDECOR distribution, with $(n_1 - 1, n_2 - 1)$ degrees of freedom:

$$f_F(x; \nu_1, \nu_2) = \frac{\Gamma[(\nu_1 + \nu_2)/2]}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} x^{\nu_1/2-1} \left(1 + \frac{\nu_1}{\nu_2} x\right)^{-(\nu_1+\nu_2)/2} \quad x > 0$$

By the virtue of symmetry we have: if the RV $\boldsymbol{\mathcal{F}}$ follows a Snedecor distribution $f_F(\nu_1, \nu_2)$ then the RV $\boldsymbol{\mathcal{F}}$':

$$\boldsymbol{\mathcal{F}}' = \frac{1}{\boldsymbol{\mathcal{F}}} = \frac{S_Y^{*2}}{S_X^{*2}}$$

follows $f_F(\nu_2, \nu_1)$; we have also some equalities of the appropriate quantiles: $\mathcal{F}(p, \nu_1, \nu_2)$ and $\mathcal{F}'(1 - p, \nu_2, \nu_1)$:

$$\mathcal{F}(p, \nu_1, \nu_2) = \mathcal{F}'(1 - p, \nu_2, \nu_1) = 1/\mathcal{F}(1 - p, \nu_2, \nu_1)$$

the so-called FISCHER TEST

With the significance level being fixed at α the critical region U_{cr} depends – as always – on the alternative hypothesis H_1:

$$\begin{aligned} H_1 : \sigma_1^2 > \sigma_2^2 \quad U_{cr} &= \Big[\mathcal{F}(1-\alpha, n_1-1, n_2-1), +\infty\Big) \\ H_1 : \sigma_1^2 < \sigma_2^2 \quad U_{cr} &= \Big[\mathcal{F}(1-\alpha, n_2-1, n_1-1), +\infty\Big) \\ H_1 : \sigma_1^2 \neq \sigma_2^2 \quad U_{cr} &= \Big[\mathcal{F}(1-\alpha/2, n_l-1, n_m-1), +\infty\Big) \end{aligned}$$

the last case corresponds to the situation when we demand the RV $\boldsymbol{\mathcal{F}}$ to be ≥ 1:

$$\boldsymbol{\mathcal{F}} = \frac{\max(S_X^{*2}, S_Y^{*2})}{\min(S_X^{*2}, S_Y^{*2})}$$

n_l and n_m are the sample size, in the numerator and in the denominator, respectively.

the FISCHER TEST