Magnetism of clustered random networks

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BASIC ELEMENTS:

Erdös-Renyi network with enhanced clusterization

±1 variables at the nodes with the Ising interaction *J* along the links

„Non-scientists tend to think that science works by deduction. But actually science works mainly by metaphor. And what's happening is that the kind of metaphors people have in mind are changing.”

[W. Brian Arthur, SFI]

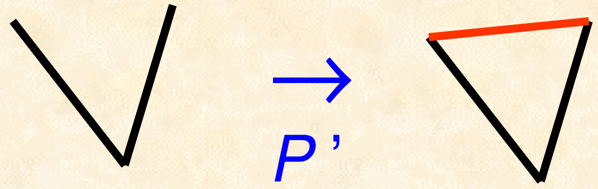
OVERVIEW

The structure: clustering, degree distribution
J>0: Curie temperature; MC *vs* mean field
J<0: Reference system: Archimedean lattice
J<0: spin glass, role of clustering

Recent review :

SN Dorogovtsev, AV Goltsev, JFF Mendes,
Critical phenomena in complex networks
arXiv:0705.0010, Pt. VI

THE STRUCTURE



The Erdös-Renyi network:

Make a link between each two nodes with probability $p_0 = \langle k_0 \rangle / N$

The enhanced clusterization*:

Make a link between each node with probability *p*’

*Method of P Holme, BJ Kim, PRE 65 (2002) 026102

The clustering coefficient *C* = probability, that two neighbours of a node are linked.

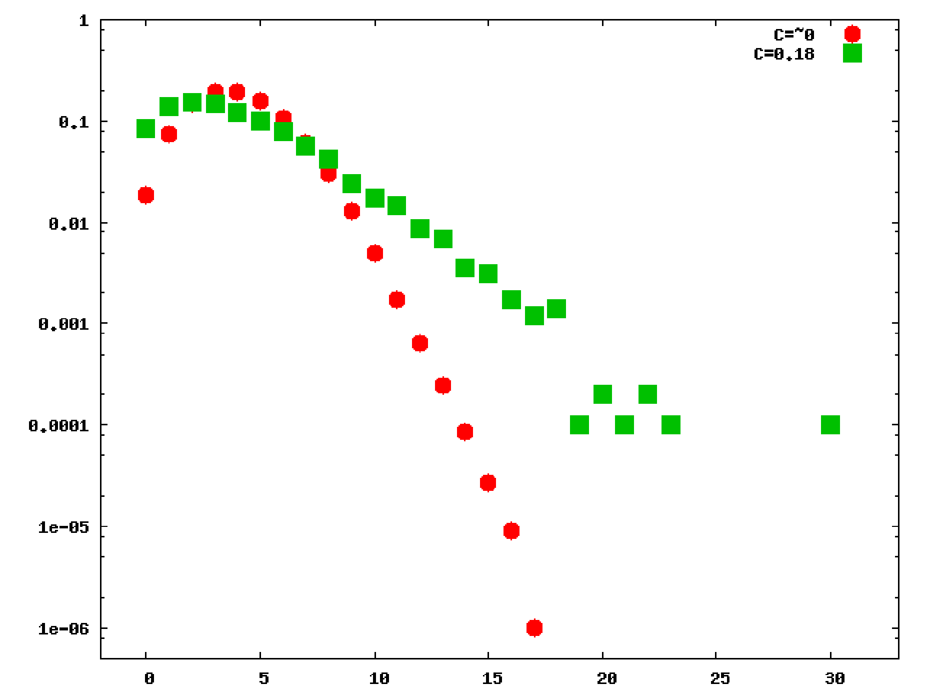
Result: $\langle k \rangle = 4.0$, $0 < C < 0.18$,

DEGREE DISTRIBUTION

C≈0, Poisson

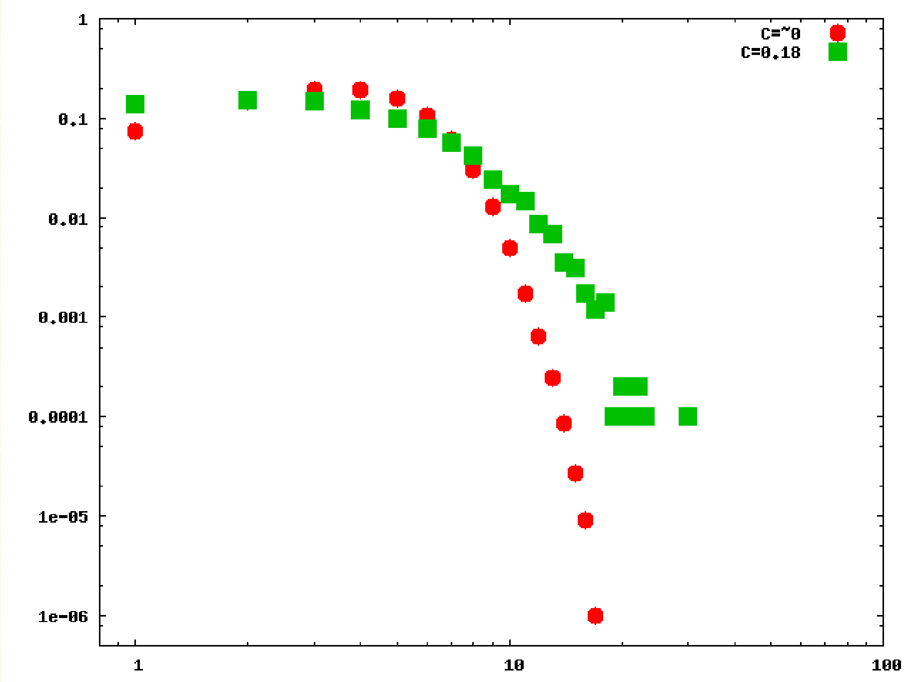
C=0.18

P(k)



k

P(k)



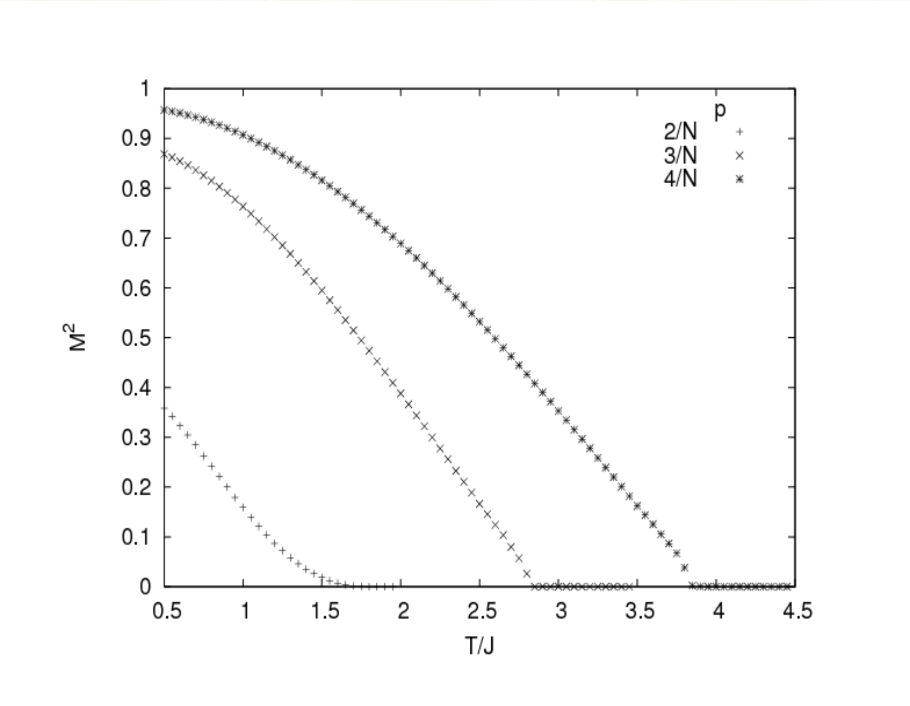
k

J > 0, CURIE TEMPERATURE

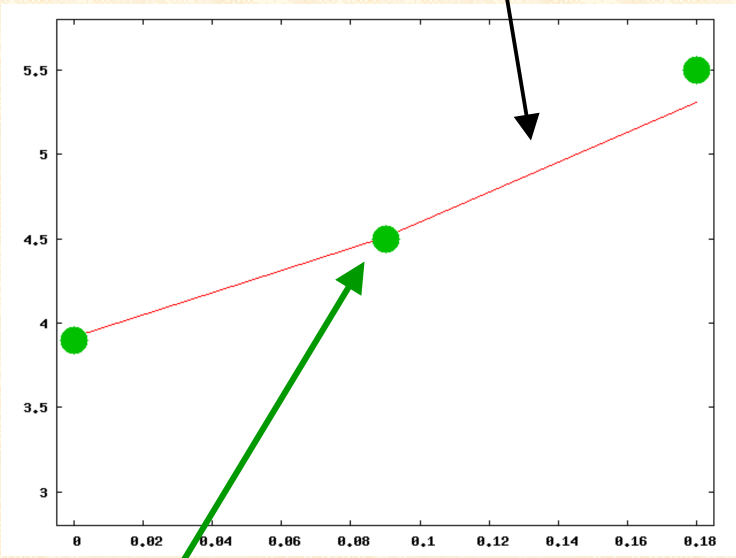
$$\frac{2J}{T_c} = \ln\frac{z_2 + z_1}{z_2 - z_1} \approx \ln\frac{<k^2>}{<k^2 - 2k>}$$

Tc

squared magnetization, C ≈ 0



temperature



Clustering coefficient *C*

MC heat bath, *N*=10^5

$J < 0$ SPIN GLASS TEMPERATURE?

I. Kanter, H. Sompolinsky, PRL 58 (1987) 164
Finite coordination number, mean-field theory, T=0
A phase diagram: $\rho(J)=\delta(J+1)$ => SG

SN Dorogovtsev, AV Goltsev, JFF Mendes, arXiv:0705.0010
For tree-like networks (weak frustration) $\rho(J)=\delta(J+1)$

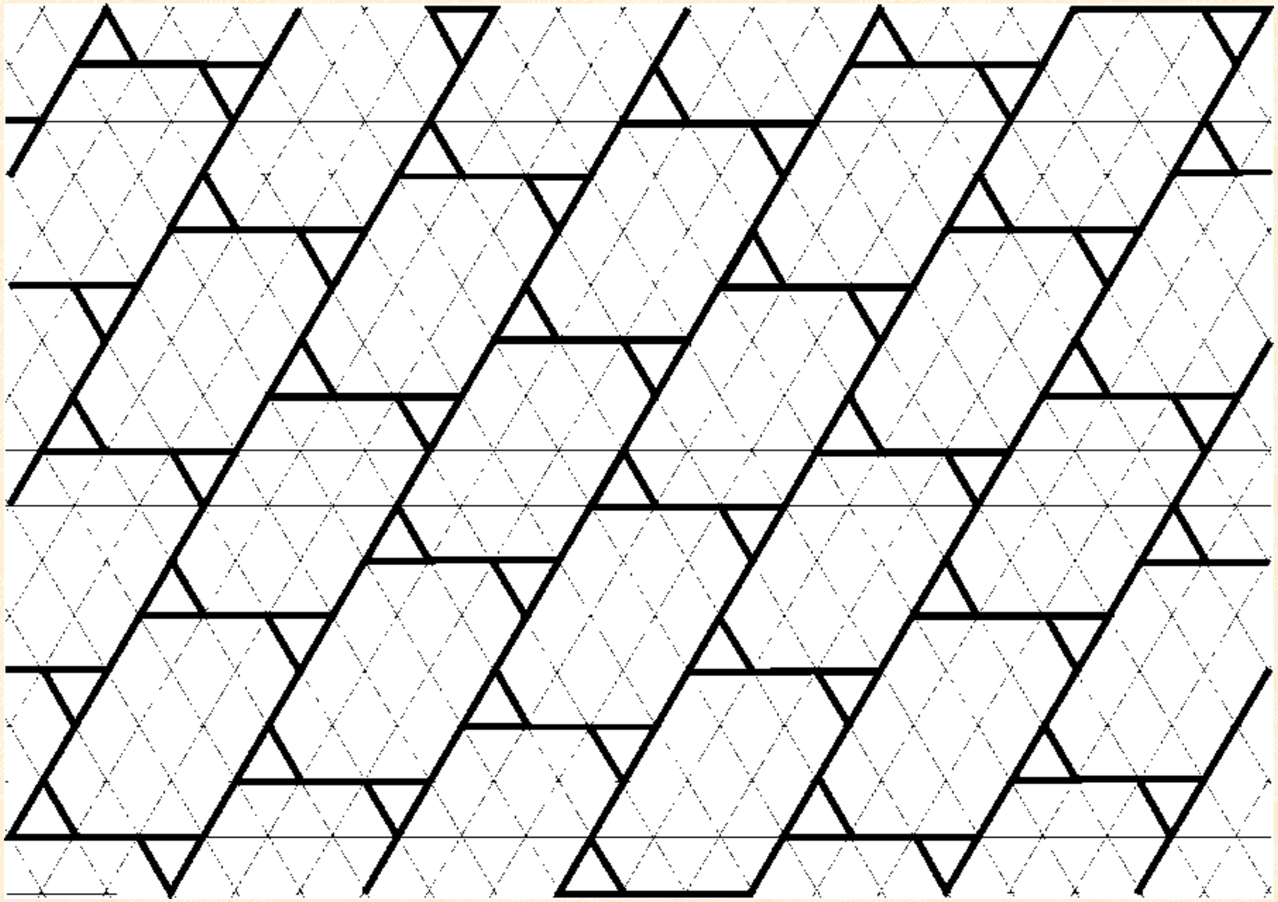
$$cth^2\beta_{SG} = cth\beta_c = \frac{z_2}{z_1} = \frac{< k(k-1) >}{< k >}$$

Then, once T_c increases with C, T_{SG} should increase as well.

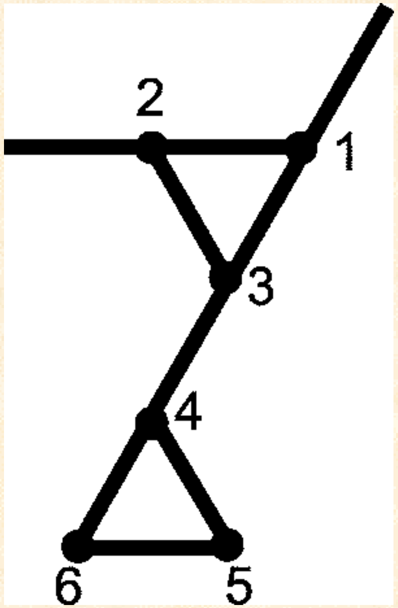
What about the influence of loops ? We have only local loops.

We have a reference system – the Archimedean lattice, where only local loops are present, too.

Reference system: stretched Archimedean $(3,12^2)$ lattice



- frustrated
- 2D
- energy gap

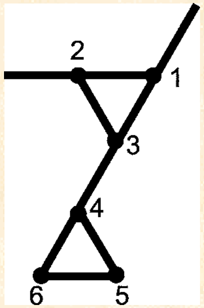


Unit cell

MJ Krawczyk e a , PRB 72 (2005) 024445

Archimedean lattice: ground state degeneracy

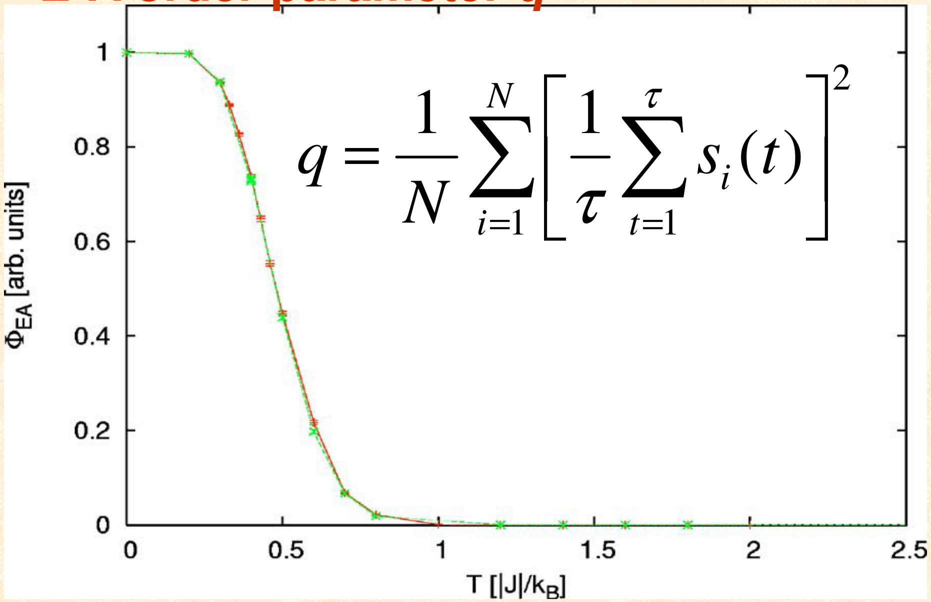
Ground state	*s*1	*s*2	*s*3	*s*4	*s*5	*s*6
A	-	+	+	-	-	+
B	-	-	+	-	+	+
C	-	+	-	+	-	+
D	+	-	-	+	+	-
E	+	+	-	+	-	-
F	+	-	+	-	+	-



Pairs which can be flipped

*s*3 , *s*4 to C or *s*2 , *s*5 to B
*s*2 , *s*5 to A or *s*1 , *s*6 to F
*s*1 , *s*6 to E or *s*3 , *s*4 to A
*s*3 , *s*4 to F or *s*2 , *s*5 to E
*s*2 , *s*5 to D or *s*1 , *s*6 to C
*s*1 , *s*6 to B or *s*3 , *s*4 to D

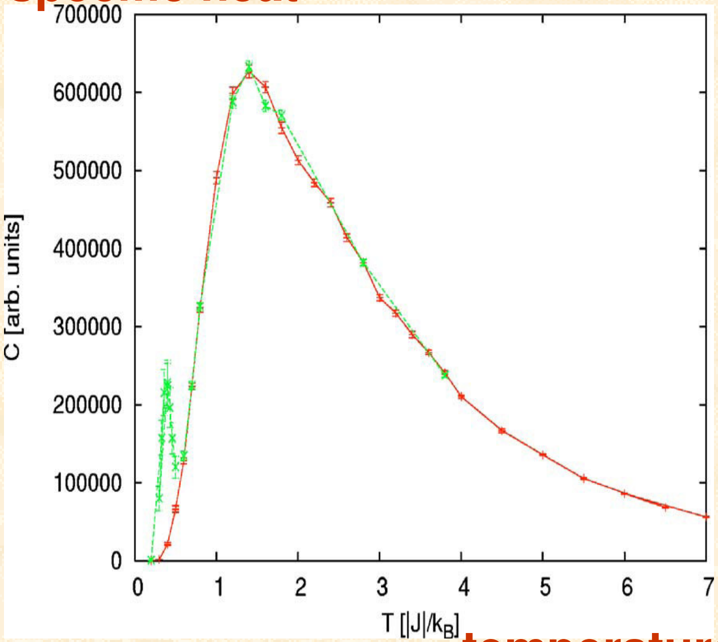
E-A order parameter *q*



$$q = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{\tau}\sum_{t=1}^{\tau} s_i(t)\right]^2$$

temperature

specific heat



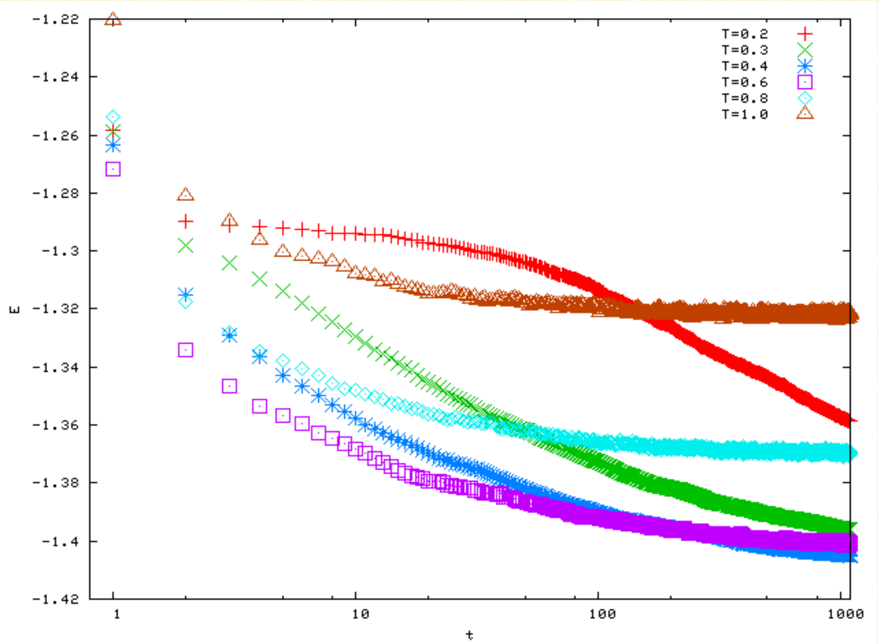
temperature

MJ Krawczyk e a , PRB 72 (2005) 024445

J < 0 ENERGY RELAXATION

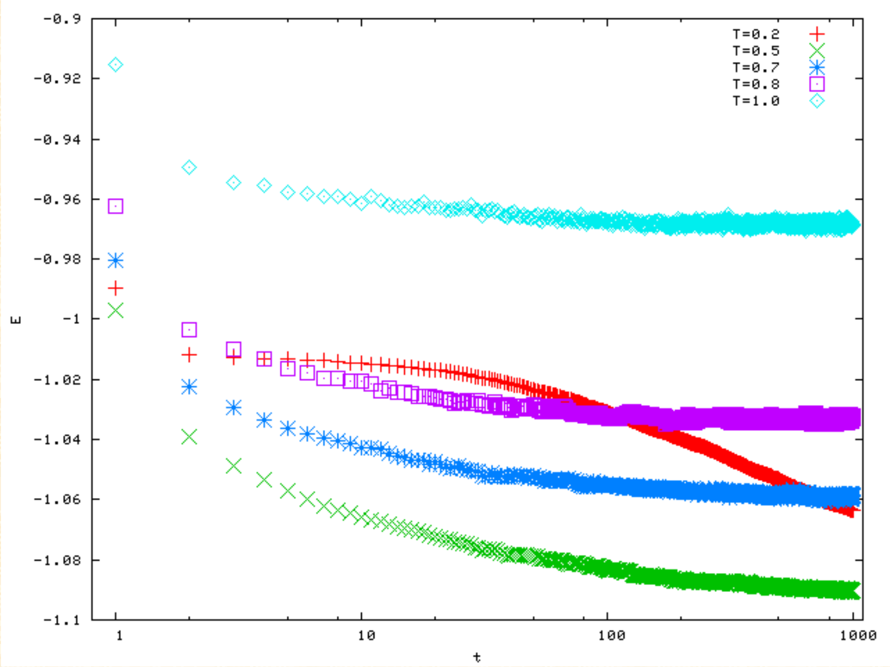
Energy

C≈0



time

C=0.18



time

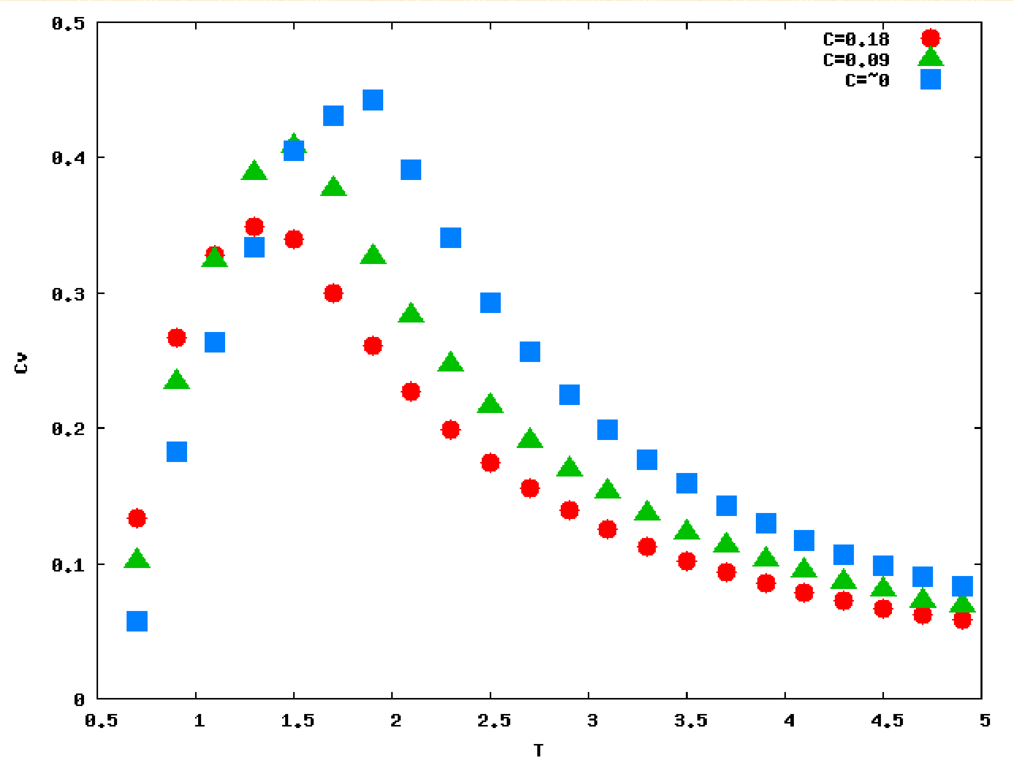
$E \approx$ const (M($t=0$))

$E \approx$ const (t) for t>1000, T > 0.7

$J < 0$

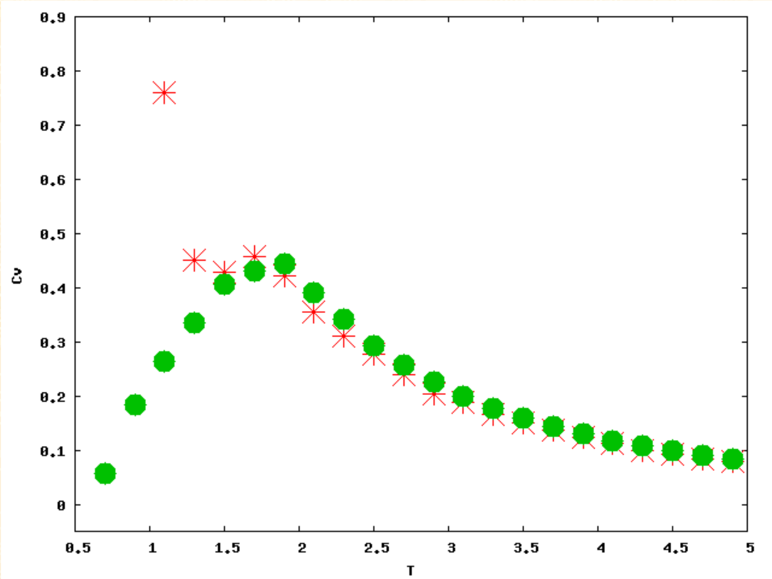
SPECIFIC HEAT C_V

$C_V = dU/dT$



temperature

$C_V = \beta\sigma_E^2$, dU/dT



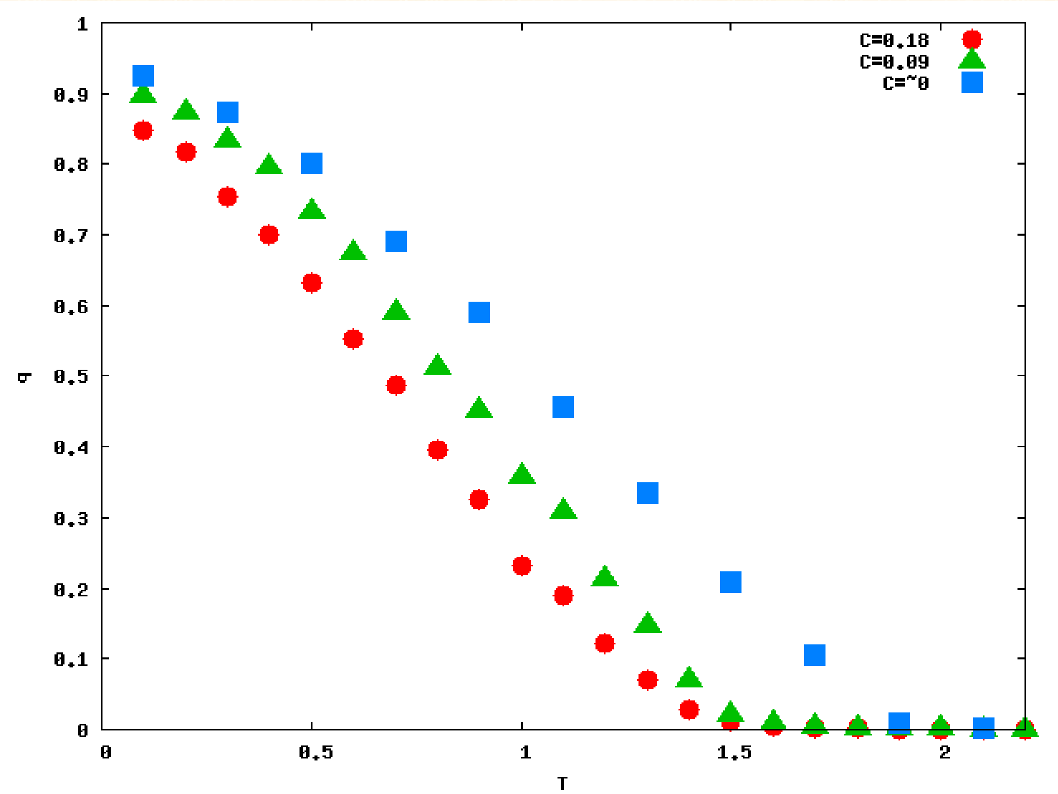
temperature

***J* < 0**

E-A ORDER PARAMETER *q*

$$q = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{\tau}\sum_{t=1}^{\tau}s_i(t)\right]^2$$

q



temperature

..whereas for the tree-like networks

C	T_{SG}
0	1.82
0.09	1.97
0.18	2.17

CONCLUSIONS

- For J>0, consequences of the enhancement of T_c with the clustering coefficient C reduce to the influence of the modification of $<k^2>$.
- For J<0, the density of frustrations increases with C, and the para–SG transition temperature decreases.

Further steps:

- the correlation functions *vs* T, C
- $\delta(k)=3$ (energy gap)
- ?...

Links to other areas:

- solid state physics: degeneracy, spin glass>spin liquid?
- computation theory: MAX-CUT, satisfiability,…
- social networks: clustered, small world, >directed
- game theory: network congestion game
- updating order > temporal networks?