Star-clique transformation and degree correlations in networks

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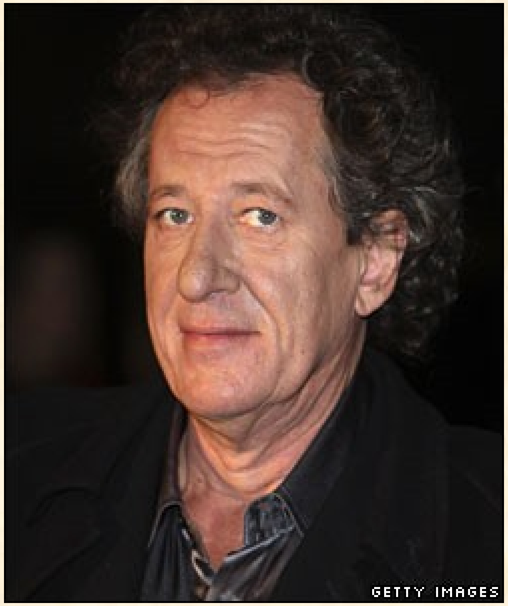
Summer Solstice Gdansk, Poland, 22-24 June 2009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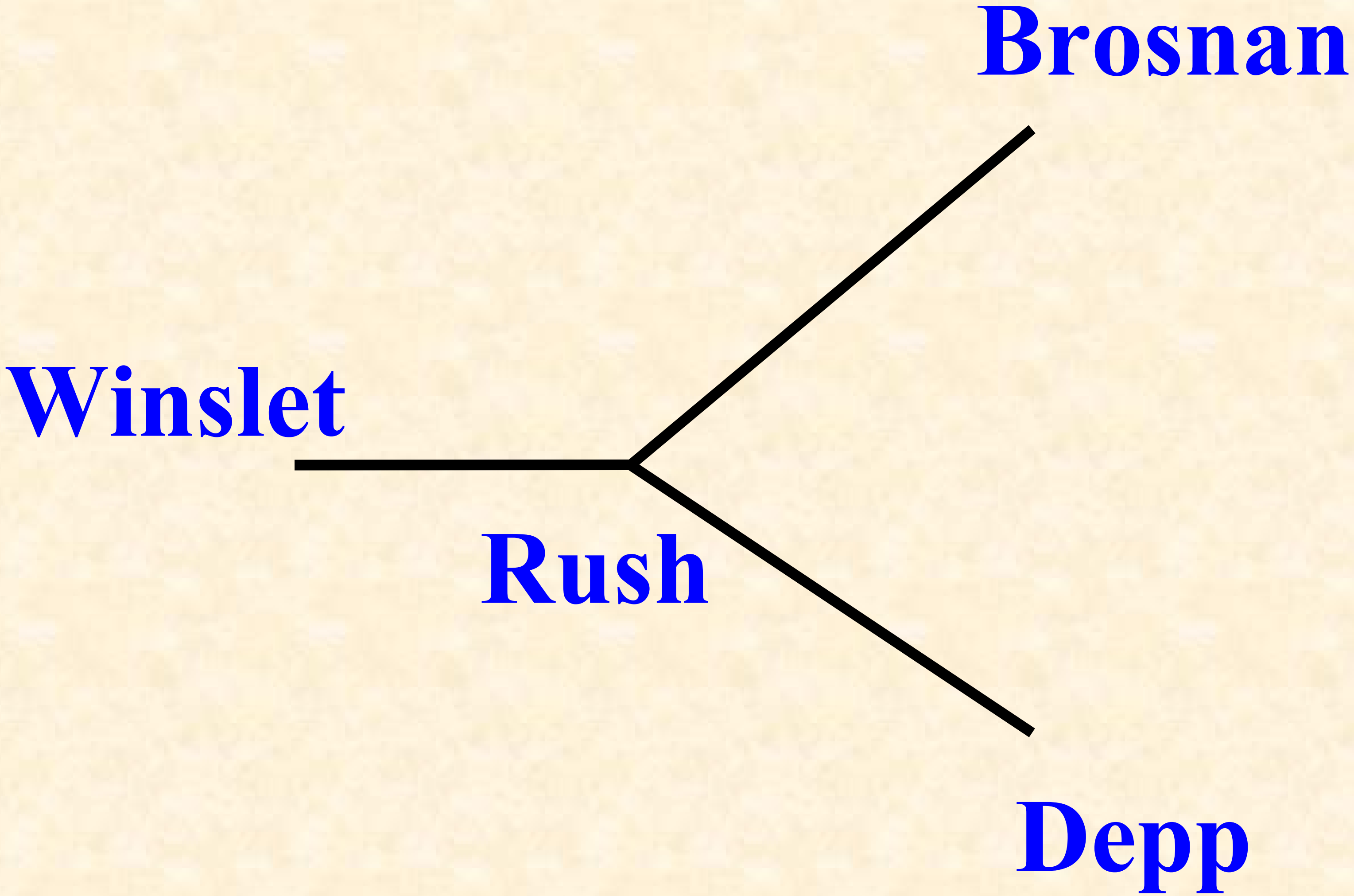
Brosnan
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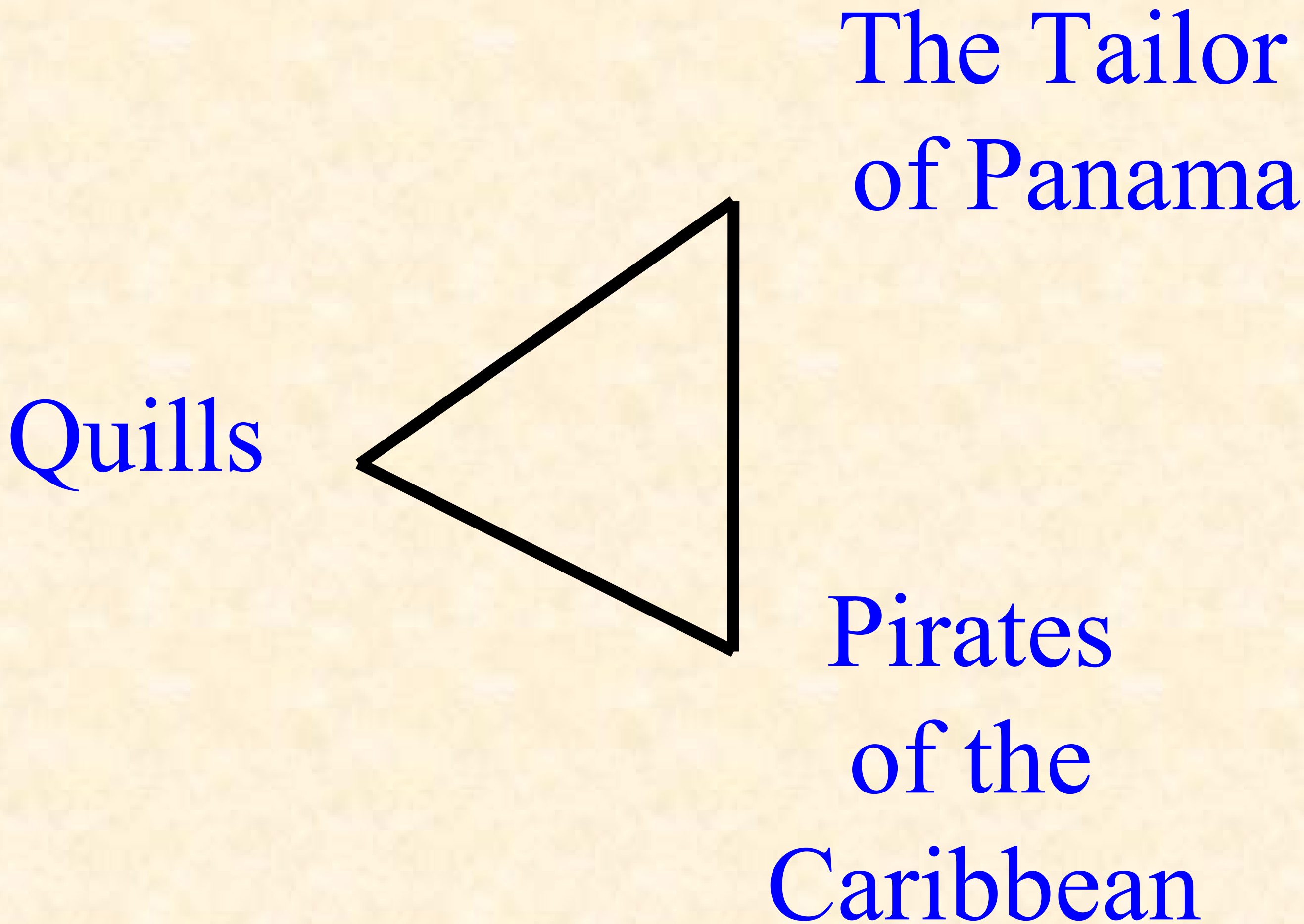

The Tailor
of Panama
Quills
Pirates
of the
Caribbean

The transformed graph is an edge graph* of the initial one.

*J.A. Bondy and U.S.R. Murty,
Graph Theory with Applications, North Holland, NY 1976
www.ecp6.jussieu.fr/pageperso/bondy/books/gtwa/gtwa.html

Another example: Coauthorship network

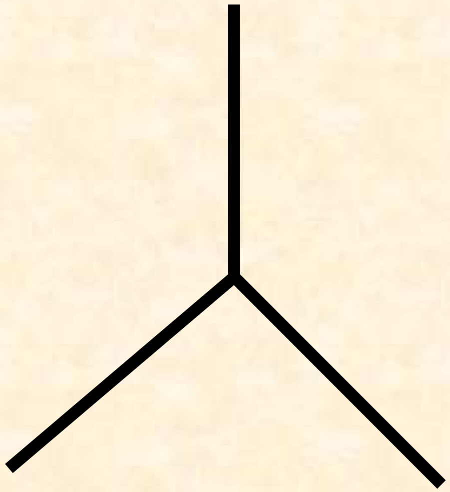
Our motivation:
communication networks, clustering**

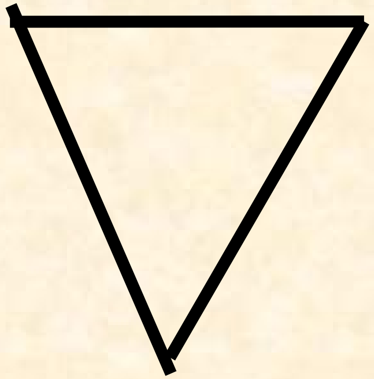
**P. Holme and B. J. Kim, PRE 65 (2002)
K. Klemm and V. M. Eguiluz, PRE 65 (2002)
M. A. Serrano and M. Boguna, PRE 72 (2005)

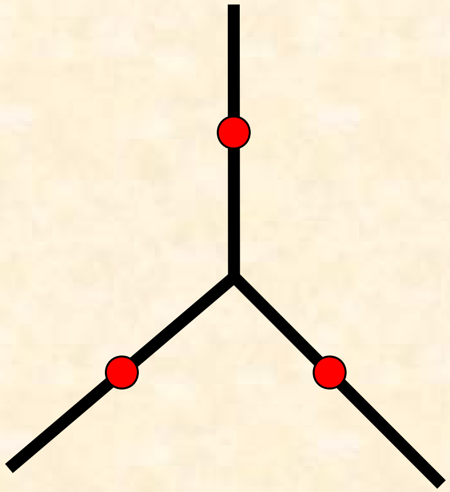
outline

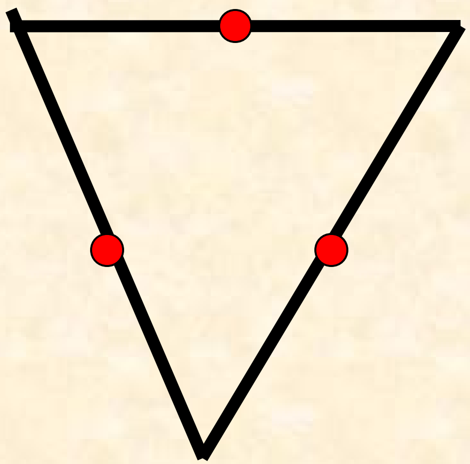
- Simple examples
- Degree distribution in transformed networks
- Analytical vs simulation results
- Clustering coefficient
- Degree-degree correlations

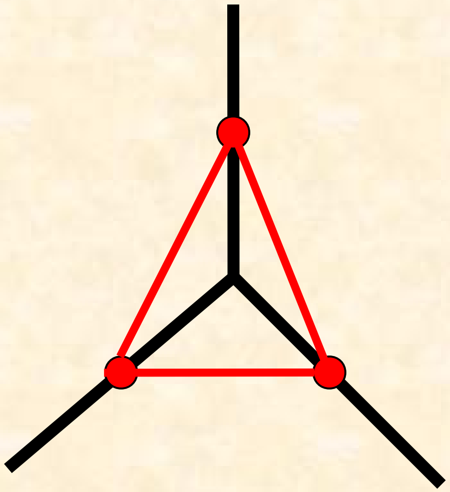
[arXiv:0904.0659]

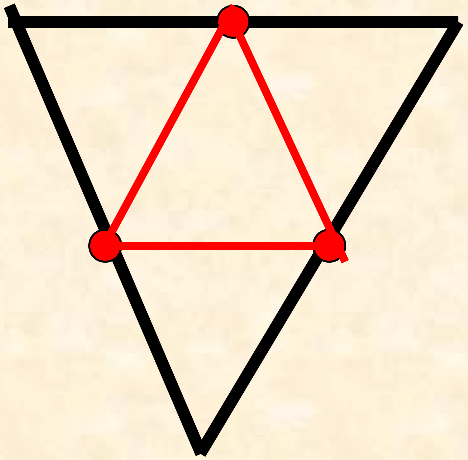


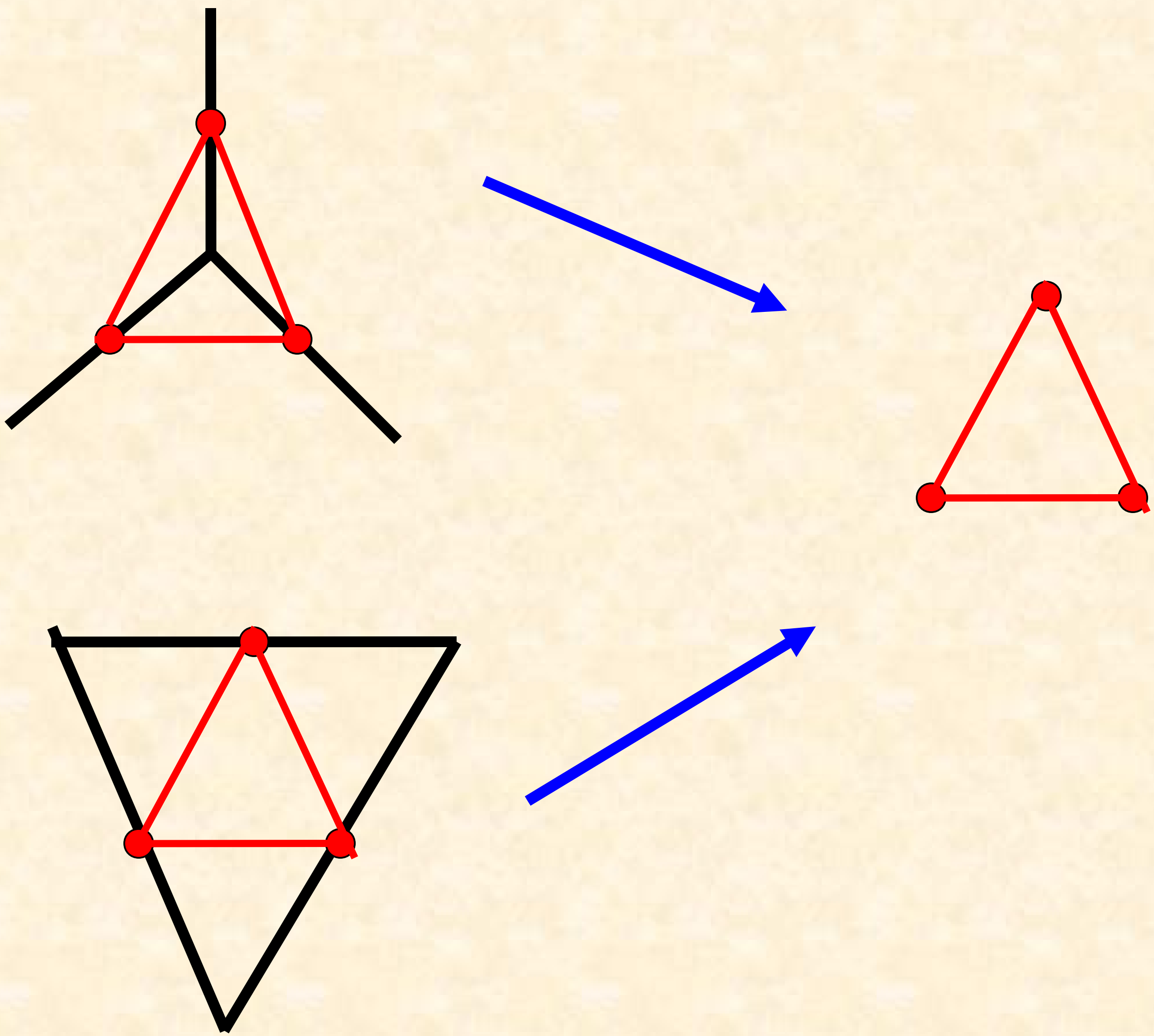


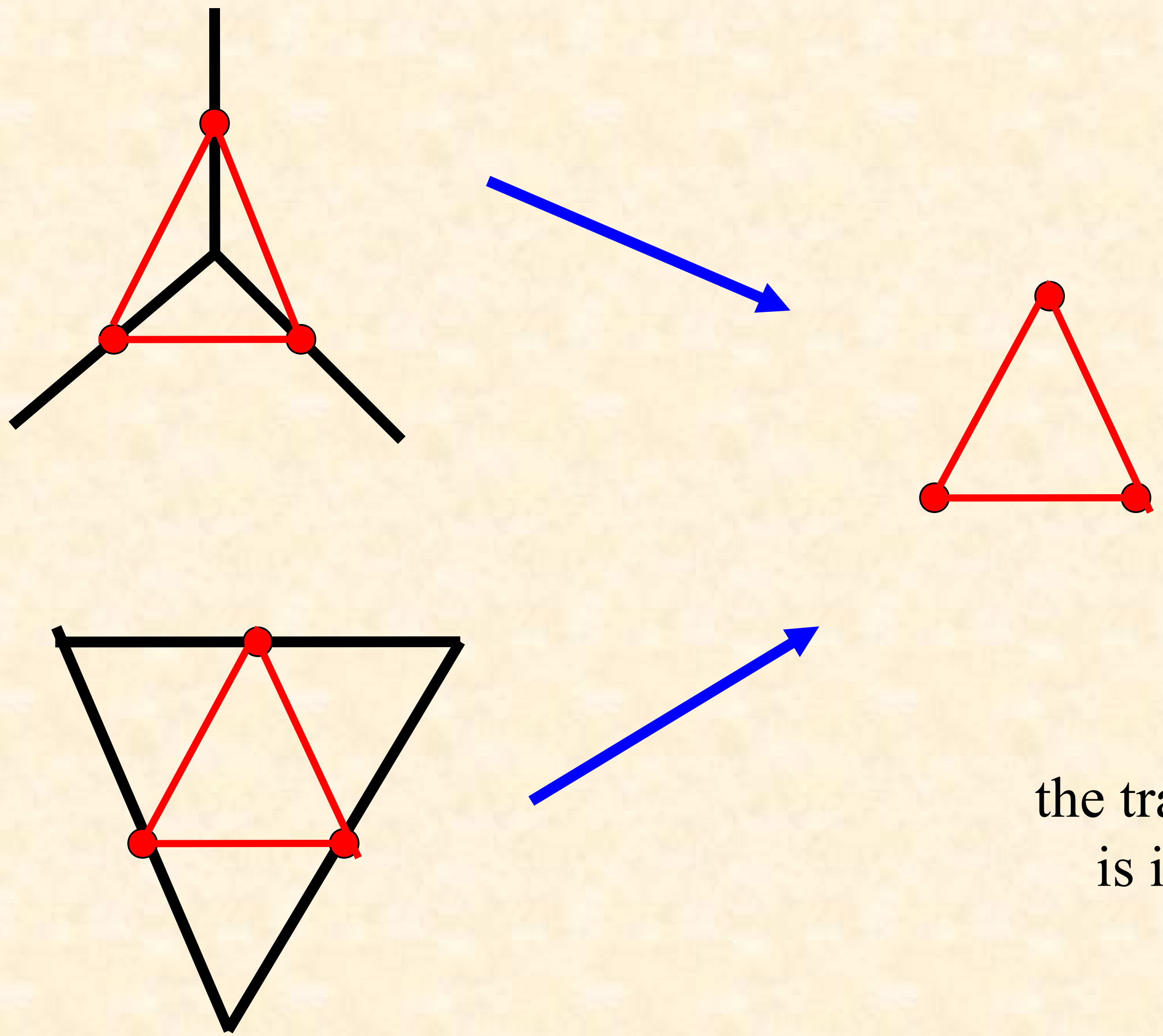




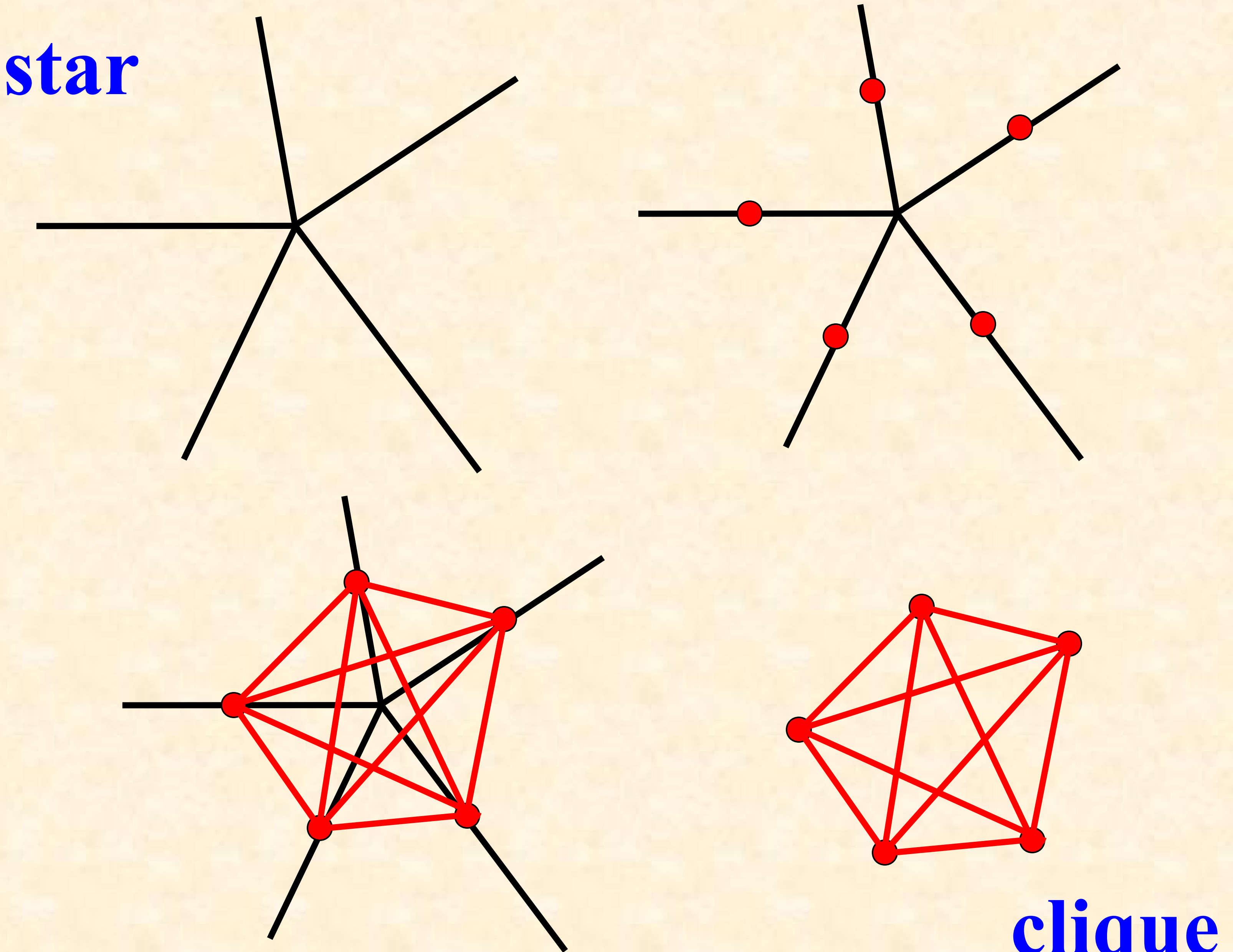




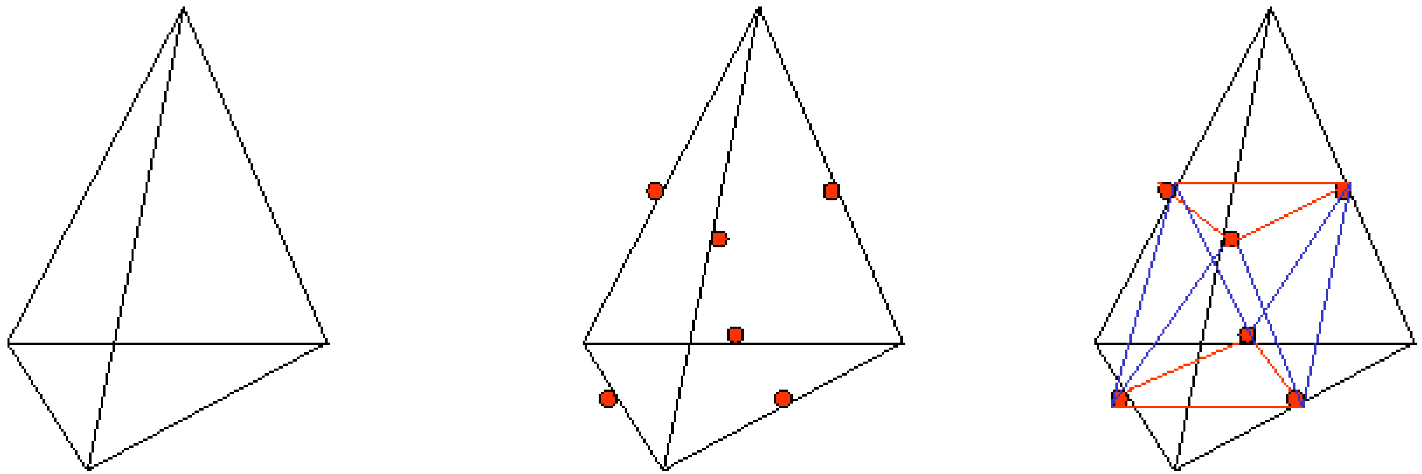


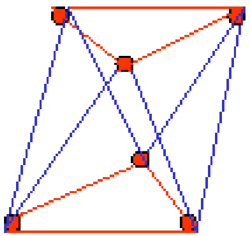


the transformation
is irreversible


star
clique

tetrahedron





octahedron

Algorithm:

1. assign numbers to the elements of the connectivity matrix above the diagonal

$$\begin{matrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 3 & 0 \\ 0 & 3 & 0 & 4 \\ 2 & 0 & 4 & 0 \end{matrix}$$

2. If *i,j* are in the same row or column, then an element *C(i,j)* of the transformed matrix is 1

$$\begin{matrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{matrix}$$

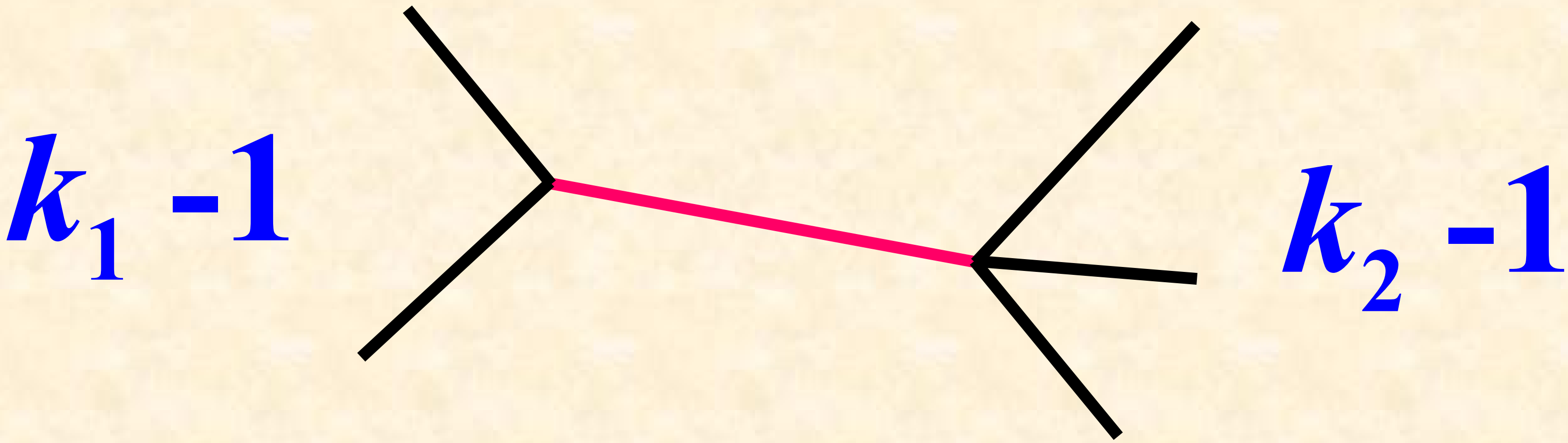
fully connected graph of N nodes ⇒

⇒ graph of $N(N-1)/2$ nodes, each with $2(N-2)$ neighbours, $C=(N-2)/(2N-5)$

graph of N nodes, each with $k<<N$ neighbours ⇒

⇒ graph of $kN/2$ nodes, each with $2(k-1)$ neighbours, $C=(k-2)/(2k-3)$

The degree distribution



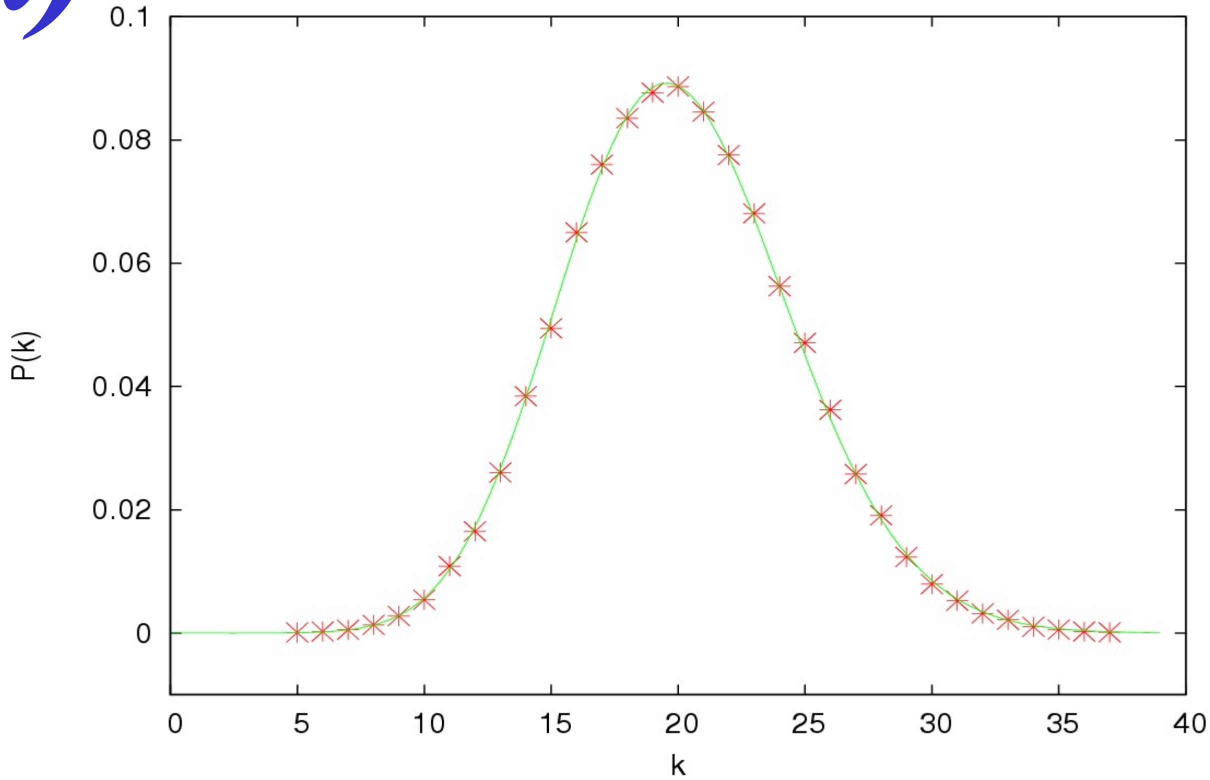
$$P_t(k) = \sum_{k_1,k_2} k_1 k_2 P(k_1) P(k_2) \delta_{k,k_1+k_2-2} =$$

$$= \sum_{k_1=1}^{k+1} k_1 (k - k_1 + 2) P(k_1) P(k - k_1 + 2)$$

Erdös-Rényi network

$$P_t(k) = e^{-2\lambda}\,\frac{(2\lambda)^k}{k!}$$

$P_t(k)$

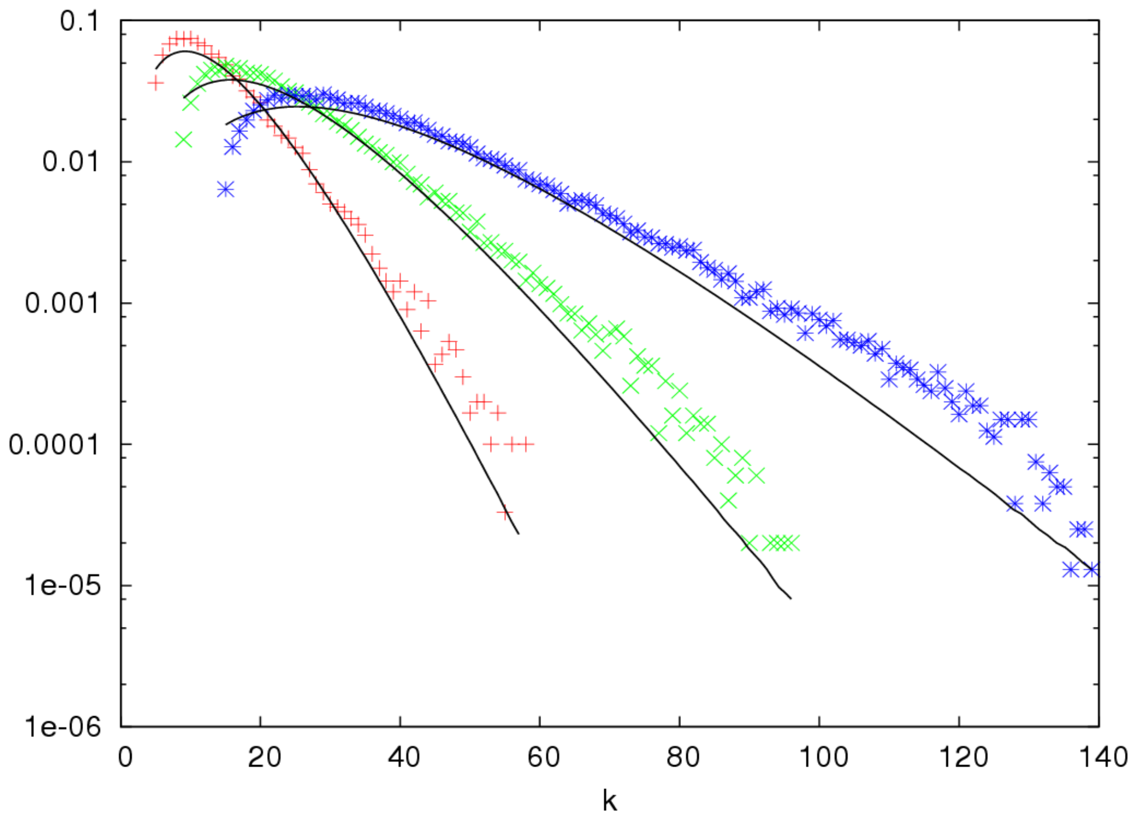


k

Exponential network

$$P_t(k) = \frac{(1-c)^4}{6}(k+1)(k+2)(k+3)c^k$$

$P_t(k)$

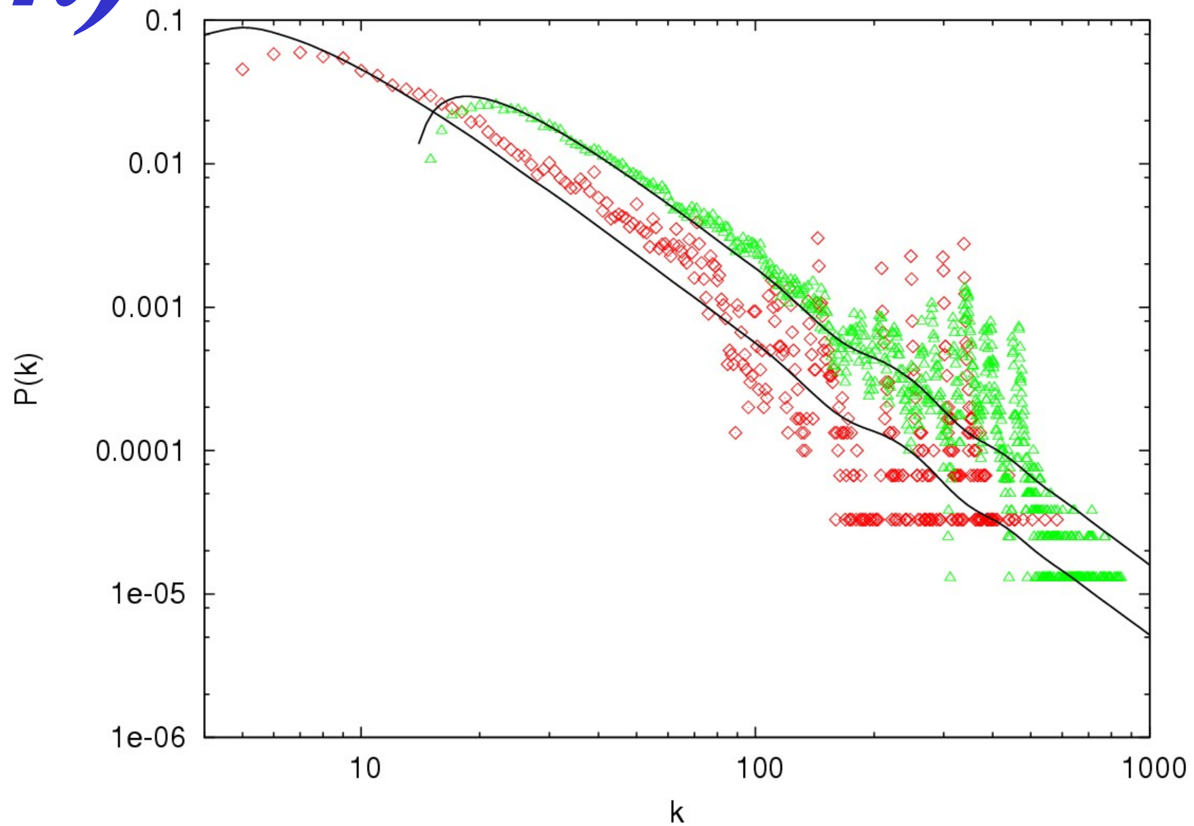


k

***M*=3,5,8**

Scale-free network

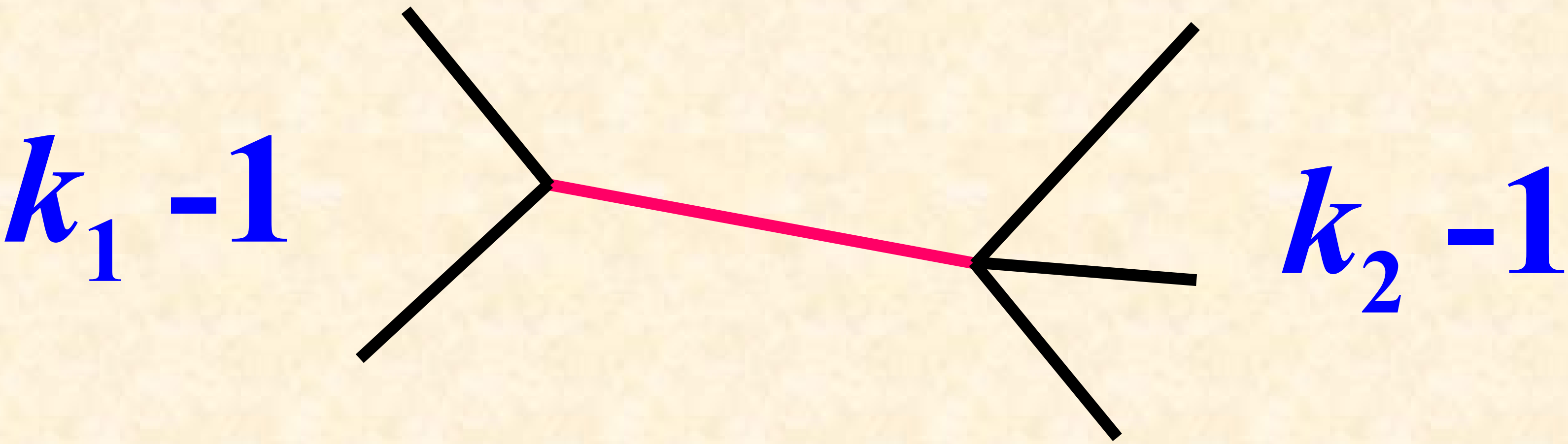
$P_t(k)$



k

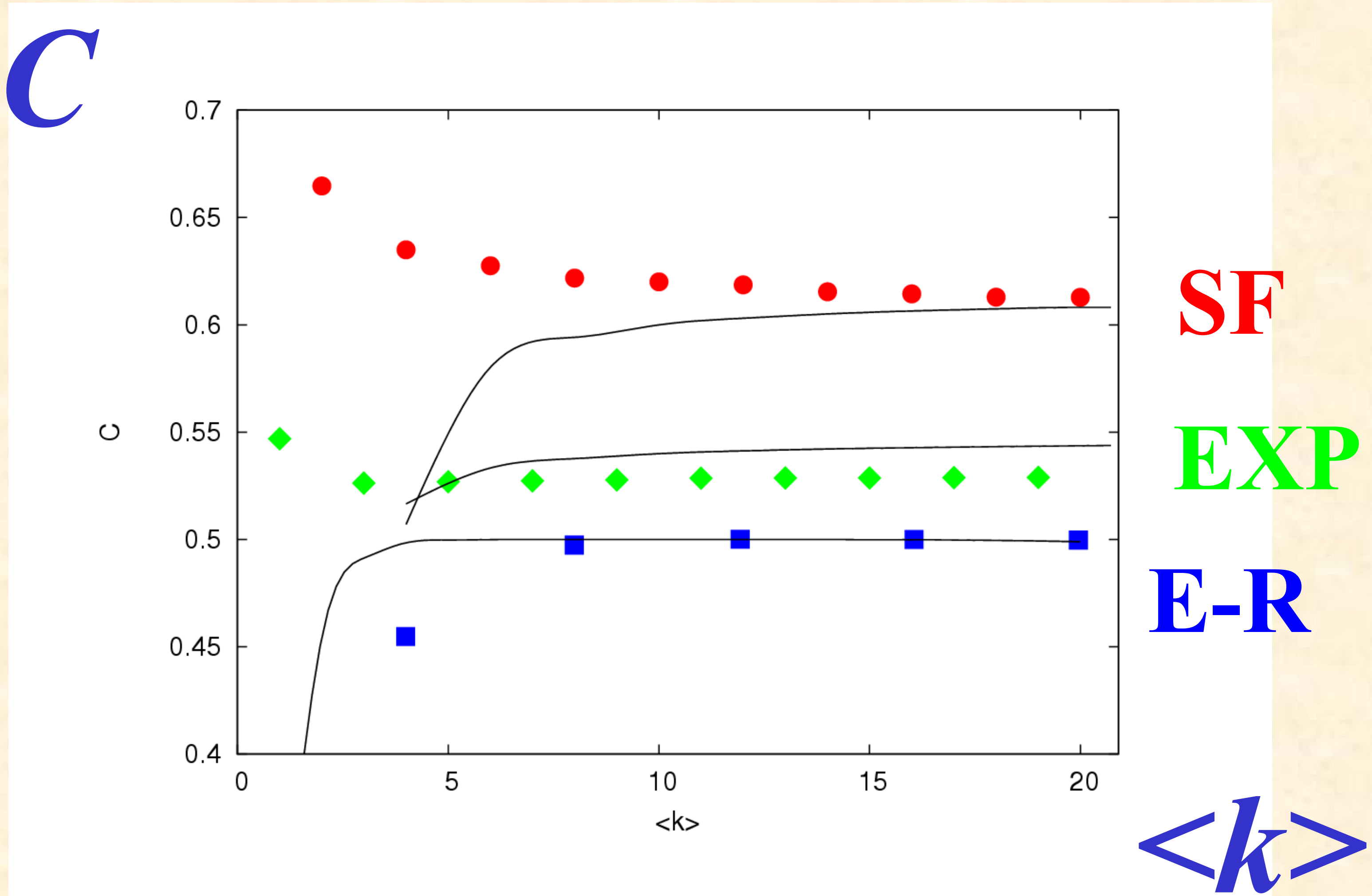
M=3,8

The clustering coefficient C



$$C = \sum_{k_1, k_2} k_1 k_2 P(k_1) P(k_2) \frac{(k_1 - 1)(k_1 - 2) + (k_2 - 1)(k_2 - 2)}{(k_1 + k_2 - 2)(k_1 + k_2 - 3)}$$

The clustering coefficient



Degree-degree correlations - are they negligible?

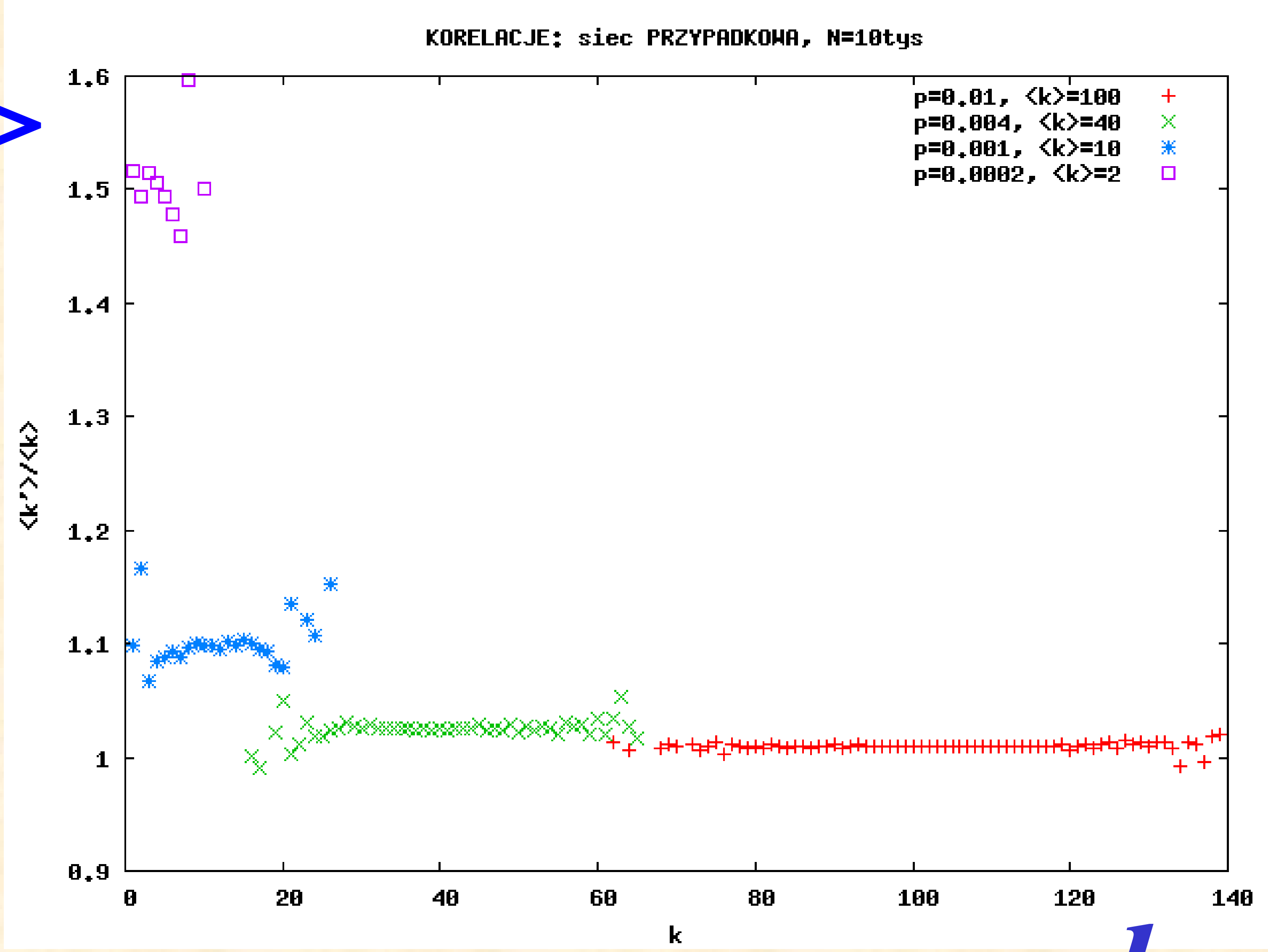
Then, the mean degree <*k'*> of neighbours of nodes of degree *k* should not depend on *k*.

$$< k' > = \sum_{k'} k' P(k' | k)$$

[A. Barrat, R. Pastor-Satorras, PRE 71 (2005) 036127.

Erdös-Rényi network

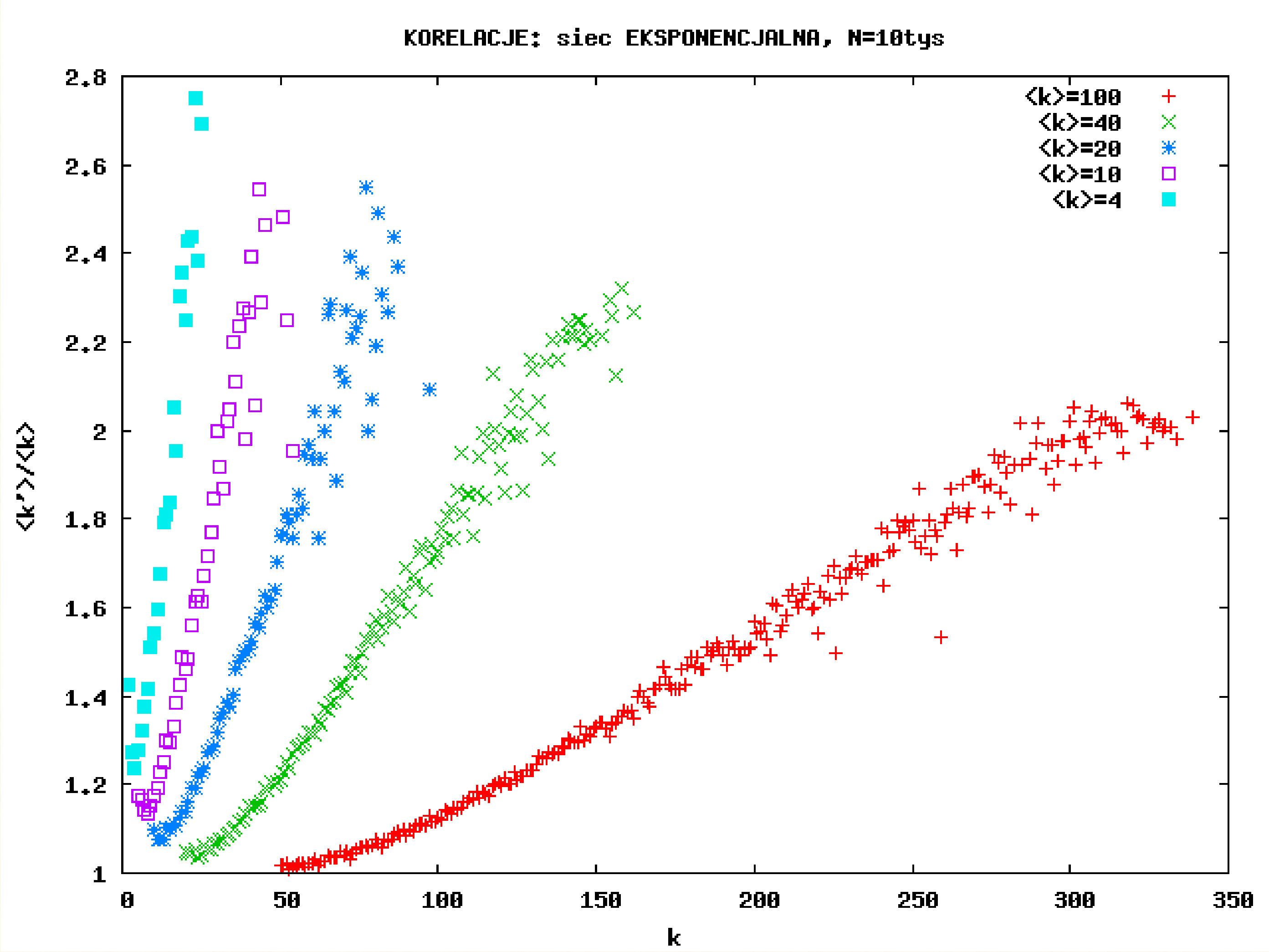
<k'>



k

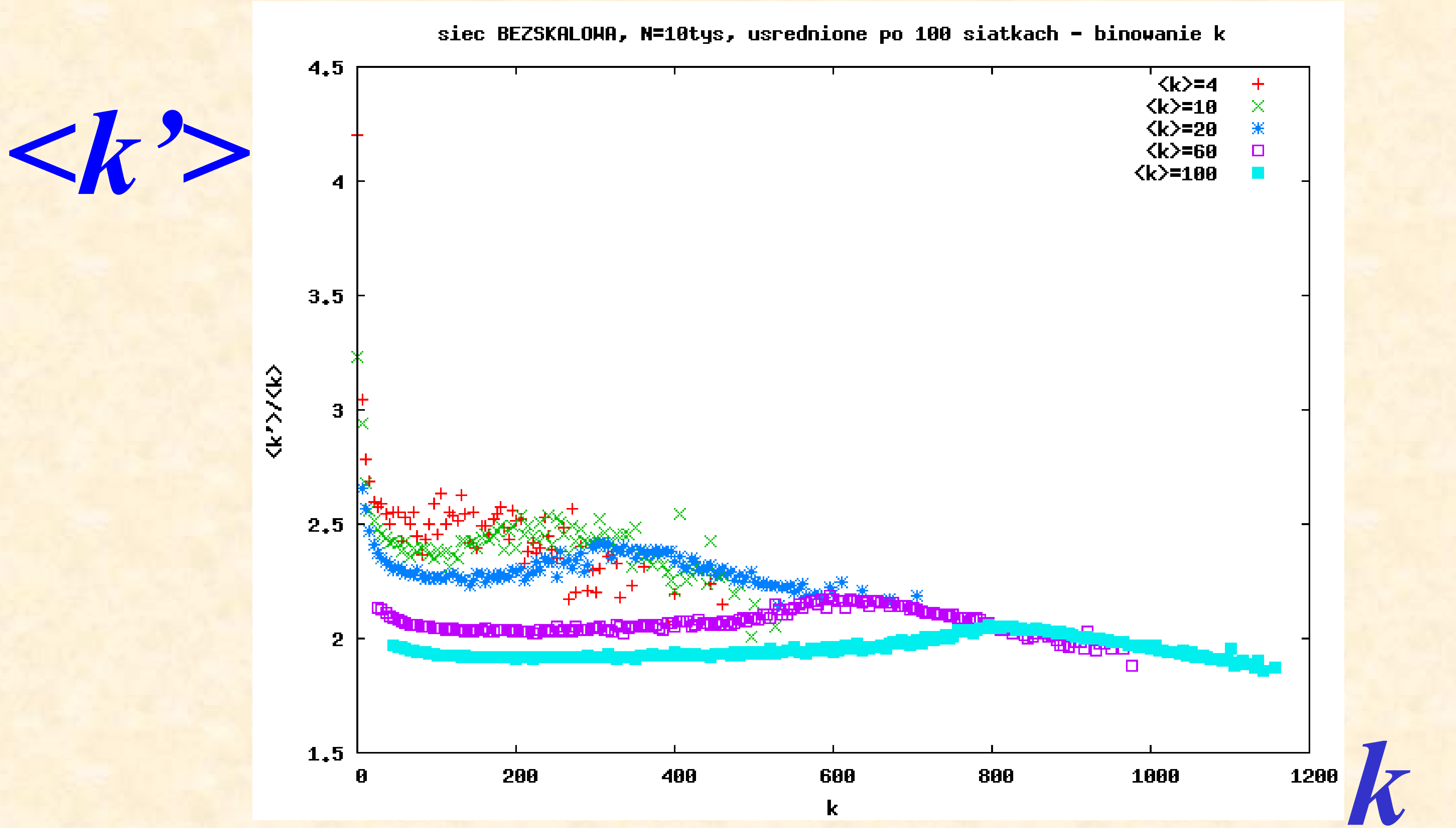
Exponential network

<k'>



k

Scale-free network



summary

In the analytical calculations of the clustering coefficient *C* the degree-degree correlations are neglected.

Differences between the simulated and analytical values of *C* are the largest for the exponential networks.

The degree-degree correlations are the largest for the exponential networks.

THANKS to Zdzisław Burda for useful references

THANK YOU FOR YOUR ATTENTION